Compressed Sensing Based Channel State Feedback for Cooperating MIMO–OFDM Systems

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***Abstract*— A remote channel estimation scheme for multiuser MIMO OFDM systems, that based on compressed sensing, is proposed. This feedback scheme significantly reduces the feedback load and the complexity at the receiver site. Extensive simulations show significant advantages of the proposed scheme over conventional feedback schemes.**

I. Introduction:

It is well known that channel state information at the transmitter (CSIT) plays a crucial role for precoding and interference mitigation in the downlink of a wireless communication system [1]. Accurate CSIT is essential to support coordinated or coherent transmission from multiple transmitters to multiple receivers. Information about the channel characteristic is usually assumed to be fully available at the receiver and "only" needs to be quantized and fed back to the transmitters. But, what if the receivers are not capable to perform channel estimation at the desired high accuracy? Channel estimation in a MIMO–OFDM system is a non–trivial task that requires significant effort and computing complexity at the receiver. In other words, can channel information efficiently be fed back without performing a complex estimation of the multi–path channel components at the receiver?

In this paper we propose a feedback, quantization and remote channel estimation scheme at the transmitter side, which does not require channel estimation at the receiver. The feedback scheme is based on the idea of compressed sensing and uses the sparsity, inherent to the overall multi–path channel, between different transmitters and a particular receiver. This includes the sparsity in the delay taps of the impulse response as well as the fast–decaying overall power distribution along different transmitters, according to their relative position. Additionally, the approach implicitly embodies adaptive compression, that is, transmitters with stronger channels are accounted with more bits in the feedback message than the ones with weaker channels. Once the transmit setup is fixed the receivers can be informed of the effective channels through dedicated pilots.

Compressed sensing techniques significantly reduce the number m of observations to be taken under the assumption that a vector of p unknown parameters has small support of size s (is s–sparse) and even if it is only compressible (sufficient decay in amplitudes). Compressed sensing has been investigated for channel estimation at the receiver side for example in [2], [3], [4], [5]. Here, the new idea is that the receiver only assists the transmitters in their estimation procedure by providing a feedback message of B bits. Hence, the remote estimation approach is a combination of sparse sampling and quantization at the receiver and recovery of the channel information at the transmitters. Such a combined setting is relative new area of research, see for example [6].

II. System Model

We consider the downlink of wireless system comprising of T transmitters and U receivers. Each transmitter is equipped with n_t transmit antennas and each receiver is equipped with n_u receive antennas. Orthogonal frequency division multiplexing (OFDM) is used to divide the available bandwidth in K orthogonal subcarriers. The signal received by receiver u on subcarrier k is given by

$$r_u(k) = (\boldsymbol{u}_u(k))^H \sum_{t=1}^{T} \boldsymbol{H}_{u,t}(k)\boldsymbol{s}_t(k) + \boldsymbol{n}_u(k),$$

where we defined the receive filter used by receiver u as $\boldsymbol{u}_u \in \mathbb{C}^{n_u \times 1}$, the channel matrix between receiver

u and transmitter t as $\boldsymbol{H}_{u,t} \in \mathbb{C}^{n_u \times n_t}$, the signal transmitted by transmitter t as $\boldsymbol{s}_t \in \mathbb{C}^{n_t \times 1}$ and the filtered noise $\boldsymbol{n}_u = \boldsymbol{u}_u^H \boldsymbol{n}$, where $\boldsymbol{n}$ is additive Gaussian noise with zero mean and unit variance. Basically, a single transmission is performed in four steps.

1) Channel measurement: Orthogonal common pilots are broadcasted by the transmitters and used by the receivers to measure the channel coefficients on certain subcarriers $\mathcal{F}$. User u stacks all measured channel coefficients in a vector $\bar{\boldsymbol{H}}_u(\mathcal{F}) \in \mathbb{C}^{|\mathcal{F}|}$. The pilots are placed uniformly in the frequency band with a spacing determined by the Nyquist criterion, such that the vector $\bar{\boldsymbol{H}}_u(\mathcal{F}) = \boldsymbol{W}\boldsymbol{h}_u$ contains all information about the vector $\boldsymbol{h}_u \in \mathbb{C}^{p'}$, which includes all complex channel coefficients between all receive antennas of receiver u and all transmit antennas of all T transmitters. The matrix $\boldsymbol{W}$ is the corresponding sub–matrix of a block DFT matrix. For each link l (i.e. each pair of receive transmit antennas) there is a corresponding block $\boldsymbol{W}_l$ on the diagonal of $\boldsymbol{W}$ which is obtained from a DFT matrix by taking only the rows $f_p \in \mathcal{F}_l \subset \mathcal{F}$ (f_p is the subcarrier index for the p'th pilot of link l) and columns $\tau_n \in \mathcal{T}_l$ ($\mathcal{T}_l$ is the sampling set of link l and τ_n is the position of the n'th sample of the channel impulse response)

$$(\boldsymbol{W}_l)_{pn} = e^{i2\pi\tau_n f_p/K}. \tag{1}$$

Usually, the sets $\mathcal{T}_l \subseteq [0 \dots \text{CP}-1]$ are determined by the maximum delay spread for the corresponding links l where CP is the number of samples of the cyclic prefix. For simplicity we will use here $\mathcal{T}_l = [0 \dots \text{CP}-1]$ for all links l.

2) Feedback: The measured channels are quantized and fed back to the transmitters as described in the next section. After decoding the feedback message transmitter t knows the estimated channels $\hat{\boldsymbol{H}}_{u,t}(k)$ for all u, k.

3) Scheduling: Based on the available CSIT a central scheduling unit selects receivers for transmission and computes for each selected receiver a precoding vector. In the sequel, the selected receivers are collected in the index set $\mathcal{S}$. The signal transmitted by transmitter t on subcarrier k is given by linear precoding, i.e.

$$\boldsymbol{s}_t(k) = \boldsymbol{V}_t(k)\boldsymbol{d}_t(k),$$

where $\boldsymbol{V}_t(k) \in \mathbb{C}^{n_t \times |\mathcal{S}|}$ is the precoding matrix used at transmitter t and $\boldsymbol{d}_t(k) \in \mathbb{C}^{|\mathcal{S}| \times 1}$ is the vector of data symbols for the selected receivers $\mathcal{S}$. The transmitted signal must satisfy a power constrained such that $\|\boldsymbol{s}_t(k)\|_2 \leq P$, where we assume uniform power allocation between different receivers.

4) Measure effective channels: Dedicated pilots (i.e. precoded pilots) are transmitted by the transmitters and used by the receivers to measure the effective channels and to compute receive filters.

III. CSI Feedback Based on Compressed Sensing

The idea of using the sparsity, inherent to the multi path channel, in the channel estimation procedure has been widely investigated for channel estimation in time–invariant (see or example [2], [4]) and the doubly–dispersive setting as well [3]. Compressed sensing based channel estimation in the LTE context was presented by the authors in [5]. Here, we present a CSI feedback scheme that is based on compressed sensing. The main idea is to feed back certain information such that channel estimation can be performed at the transmitters. In this section we consider an arbitrary receiver u and drop the receiver index for notational convenience.

A. Proposed Remote Channel Estimation

The goal is to inform the cooperating transmitters about the real vector $\boldsymbol{x} := [\text{Re}\{\boldsymbol{h}\}, \text{Im}\{\boldsymbol{h}\}] \in \mathbb{R}^p$ ($p = 2p'$) which admits sparsity (or is "compressible" in a more general notion), i.e. only a small fraction $s \ll p$ of p components are nonzero (or the vector ordered by amplitudes has fast decay). To achieve this goal, the real vector $[\text{Re}\{\bar{\boldsymbol{H}}(\mathcal{F})\}, \text{Im}\{\bar{\boldsymbol{H}}(\mathcal{F})\}]$ is multiplied at the receiver by a so called "measurement matrix" $\boldsymbol{\Psi} \in \mathbb{R}^{m \times n}$ obeying with $\boldsymbol{\Phi} := [\text{Re}\{\boldsymbol{W}\}, -\text{Im}\{\boldsymbol{W}\}; \text{Im}\{\boldsymbol{W}\}, \text{Re}\{\boldsymbol{W}\}] \in \mathbb{R}^{n \times p}$ a restricted isometry property (RIP) [7] for the sparsity s where $n := 2|\mathcal{F}|$. The matrix $\boldsymbol{\Lambda} := \boldsymbol{\Psi\Phi} \in \mathbb{R}^{m \times p}$ is said be s–RIP if there exists $\delta_s < 1$ such that

$$(1 - \delta_s)\|\boldsymbol{c}\|_2^2 \leq \|\boldsymbol{\Lambda c}\|_2^2 \leq (1 + \delta_s)\|\boldsymbol{c}\|_2^2$$

holds for all $\boldsymbol{c} \in \mathbb{R}^p$ with $\|\boldsymbol{c}\|_0 := |\{k : c_k \neq 0\}| \leq s$ meaning that $\boldsymbol{\Lambda}$ acts almost isometrically on s–sparse vectors. For example, random matrices have, with overwhelming probability, good RIP properties and the number $m = \mathcal{O}(s \log p)$ of observations is essentially linearly in s and of logarithmic order in the ambient dimension p [8]. Conventional methods usually scale linearly in p. Deterministic constructions of RIP–matrices are known as well (see Section III-B) such that this feedback scheme can also work without common randomness.

The real vector $\boldsymbol{y} = \boldsymbol{\Lambda x}$ is quantized and fed back to the transmitters

$$\boldsymbol{y}_Q = \mathcal{Q}(\boldsymbol{\Lambda x}) \tag{2}$$

where $\mathcal{Q} : \mathbb{R}^m \to \mathcal{M}$ denotes the quantizer and $\mathcal{M}$ is the feedback message set of cardinality $|\mathcal{M}| = 2^B$. The transmitters have to solve – formally – the combinatorial problem

$$\min_{\boldsymbol{\Lambda} \cdot c \in \mathcal{Q}^{-1}(y_Q)} \|\boldsymbol{c}\|_0 \tag{3}$$

where $\mathcal{Q}^{-1}(\boldsymbol{y}_Q) := \{\boldsymbol{y} \in \mathbb{R}^m : \mathcal{Q}(\boldsymbol{y}) = \boldsymbol{y}_Q\}$ is the quantization region for the feedback message $\boldsymbol{y}_Q$. To overcome the combinatorial character of the objective function $\|\boldsymbol{c}\|_0$ its convex relaxation $\|\boldsymbol{c}\|_1 := \sum_{k=1} |c_k|$ is considered instead. For convex quantization regions (3) has then been replaced by a convex optimization problem. For simplicity we consider here mean–squared error quantization, i.e. $\mathcal{Q}^{-1}(\boldsymbol{y}_Q)$ are balls. Having available $\boldsymbol{y}_Q$ and the real matrix $\boldsymbol{\Lambda} \in \mathbb{R}^{m \times p}$ the transmitters can solve the optimization problem

$$\hat{\boldsymbol{x}} = \arg\min_{\boldsymbol{c}} \frac{1}{2}\|\boldsymbol{y}_Q - \boldsymbol{\Lambda c}\|_2^2 + \tau\|\boldsymbol{c}\|_1. \tag{4}$$

The estimate $\hat{\boldsymbol{x}}$ is usually called the *basis pursuit denoising estimate* and τ is a non–negative regularization parameter. Optimally, τ has to be adjusted according to several parameters of the system operating point. As indicated for example in [10] a reasonable choice (also for convergence behavior) is to write $\tau = c \cdot \|\boldsymbol{\Lambda}^T y_Q\|_\infty$, where $c = c(m, n, p, B, \dots)$ has to be calibrated numerically.

Problem (4) can be casted as a quadratic program on a cone by splitting the vector $\boldsymbol{c} = \boldsymbol{c}^+ - \boldsymbol{c}^-$ into its positive and negative parts $\boldsymbol{z} := [\boldsymbol{c}^+, \boldsymbol{c}^-] \geq 0$. The problem is then equivalent to $\min_{\boldsymbol{z} \geq 0} \langle \boldsymbol{d}, \boldsymbol{z} \rangle + \langle \boldsymbol{z}, \boldsymbol{\Omega z} \rangle / 2$ where $\boldsymbol{\Omega} = (\boldsymbol{\Lambda}^T \boldsymbol{\Lambda}) \otimes \begin{pmatrix} +1 & -1 \\ -1 & +1 \end{pmatrix}$ and $\boldsymbol{d} = [\tau \cdot \boldsymbol{1} - \boldsymbol{\Lambda}^T \boldsymbol{y}_Q, \tau \cdot \boldsymbol{1} + \boldsymbol{\Lambda}^T \boldsymbol{y}_Q]$ which can be solved for example with the GPSR algorithm presented in [10]. From the channel estimate $\hat{\boldsymbol{x}} = [\mathrm{Re}\{\hat{\boldsymbol{h}}\}, \mathrm{Im}\{\hat{\boldsymbol{h}}\}]$ the transmitters proceed as if they would know $\boldsymbol{h}$, i.e. perform scheduling and compute precoders.

B. Deterministic Construction of RIP Matrices

It is well known that random matrices with i.i.d. Gaussian entries have overwhelming RIP properties. However, in practical systems deterministic constructions of RIP matrices may be a huge advantage. In [9] a construction for deterministic RIP matrices of order $m = \mathcal{O}(p^{\frac{1}{2}+\alpha})$, with $\alpha > 0$ a constant, is given. The following construction of deterministic RIP matrices is given in [9]. Take ξ a large integer and m a large prime. Let

$$\mathcal{A} = \left\{1, 2, \dots, \lfloor m^{1/2} \rfloor\right\},$$

set $M = 2^{2\xi-1}$, $r = \lfloor \frac{\log m}{2\xi \log(2)} \rfloor$ and let

$$\mathcal{B} = \left\{ \sum_{j=0}^{r-1} x_j (2M)^j : 0 \leq x_j \leq M - 1 \right\},$$

then the columns of the deterministic measurement matrix $\boldsymbol{\Psi}$ are

$$\psi_{a,b} = m^{-1/2} \left(e^{2\pi i (ax^2 + bx)/m} \right)_{1 \leq x \leq m},$$

with $a \in \mathcal{A}$, $b \in \mathcal{B}$. The parameters should be chosen such that $|\mathcal{F}| = |\mathcal{A}||\mathcal{B}|$.

IV. Simulations

In this section we compare the proposed remote channel estimation feedback scheme with different transmit and feedback strategies by extensive simulations.

A. Feedback and Transmit Strategies

The proposed remote channel estimation feedback scheme, described above, is compared with the following feedback schemes.

Genie aided time domain quantization: The path position of the channel impulse response are assumed to be perfectly known to the transmitters. The real and imaginary parts of the complex channel coefficients (i.e. s real scalars) are independently quantized, i.e. IQ quantization.

IQ frequency domain quantization: The channel on each subcarrier (or a group of subcarriers) is quantized by IQ quantization.

Rate Approximation: The Rate Approximation feedback and transmit strategy was proposed by the authors in [11]. Rate Approximation is a fixed codebook scheme, i.e. the precoding vectors must be chosen from a transmit codebook. The main idea is that the receivers select a channel quantization vector from a feedback codebook $\mathcal{V}$ such that the transmitters can approximate the rates of each receiver subject to a small uniform a priori error. In the simulations the transmit codebook $\mathcal{C}$ and feedback codebook $\mathcal{V}$ are given by the LTE codebooks defined in [12].

In the simulations we consider coherent transmission from multiple transmitters to multiple receivers and uncoordinated transmission from multiple transmitters to multiple receivers.

Coherent transmission: In the coherent transmission mode each transmitter serves the same set of receivers. The precoding vectors are computed according to block diagonalization (BD) [13] using partial CSIT. BD is known to be very sensitive under CSIT uncertainties [14].

Uncoordinated transmission: In the uncoordinated transmission mode each transmitter $t = 1, \ldots, T$ serves a disjoint set of receivers $\mathcal{S}_t$. The precoding vectors are computed according to zero forcing beamforming (ZFBF). That is, transmitter t selects the precoder for receiver $u \in \mathcal{S}_t$ to lie the null space of the partial CSIT $\hat{\boldsymbol{H}}_{v,t}(k)$, $v \in \mathcal{S}_t \setminus \{u\}$.

For both transmission modes scheduling is performed in a greedy fashion. First, the best receiver is selected, subsequently additional receivers are added to the system, if this increases the (estimated) system sum rate.

B. Simulation Setup

In the simulations we consider 3 base stations (i.e. transmitters) located in 3 adjacent cells and 10 users (i.e. receivers) uniformly distributed over the network area, i.e. a radius of 250 meter around the center of the base stations. The physical layer is configured according to LTE [12]. The base station are equipped with $n_t = 4$ transmit antennas and each user is equipped with $n_u = 1$ receive antennas. The channels are modeled by the spatial channel model extended (SCME) [15] using the urban macro scenario. The simulation parameters are summarized in Table I.

TABLE I
SIMULATION PARAMETERS

Parameter	Value/Assumption
Number of transmitters	3
Frequency reuse	full
# receivers K	10 (uniformly distributed)
# transmit antennas n_t	4 (uncorrelated)
# receive antennas n_u	1
Equivalent SNR	153 dB
LTE carrier frequency	2 GHz
Bandwidth	5 MHz
LTE channel model	SCME (urban macro)
Inter cell interference	explicit modeling

C. Simulation Results

Figure 1 depicts the mean spectral efficiency over the feedback bits per feedback message per user. We make the following observations:

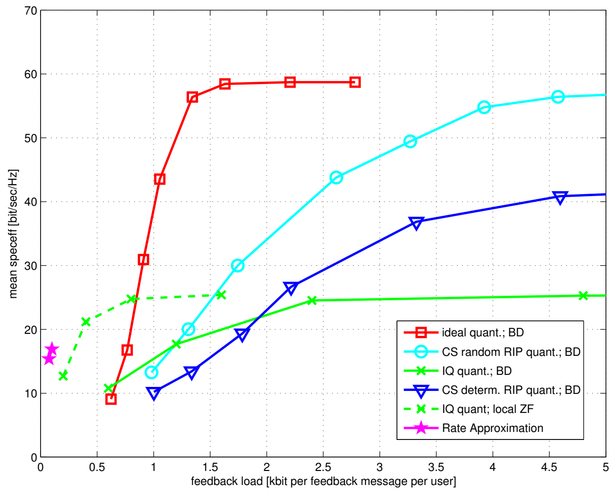

Fig. 1. Mean spectral efficiency over feedback bits. Comparing coherent transmission (i.e. BD) and uncoordinated transmission (i.e. ZFBF) for different feedback schemes. Simulation setup given in Table I.

Genie aided quantization with BD: Has the best performance with a feedback rate above 0.5 kbits per feedback message. However it would require perfect channel estimation at the receivers and a genie that provides the path positions to the transmitters. Therefore, this performance can be viewed as an upper bound.

Proposed remote channel estimation with BD: Requires more feedback bits per feedback message, then the genie aided approach, but for sufficiently high feedback rates achieves the same performance. With random measurement matrices the performance is about $10-20\%$ better than the performance with deterministic measurement matrices. Above a feedback rate of 1.5 kbit per feedback message (2 kbit for deterministic measurement matrices) coherent transmission outperforms uncoordinated transmission.

IQ channel quantization with BD: Using IQ frequency domain quantization the average channels (averaged over 12 subcarriers) are quantized. Therefore, this feedback scheme does not achieve the maximal performance of the genie aided scheme and is outperformed by the proposed remote channel estimation scheme. Without averaging the channels the feedback rates would increase by a factor of 12.

Rate Approximation: The Rate Approximation scheme is a feedback and uncoordinated transmit scheme which is optimized for very low feedback rates. We observe that with about 0.1 kbits per feedback message Rate Approximation performs as good as genie aided feedback with coherent transmission at a feedback rate of 0.7 kbits per feedback message.

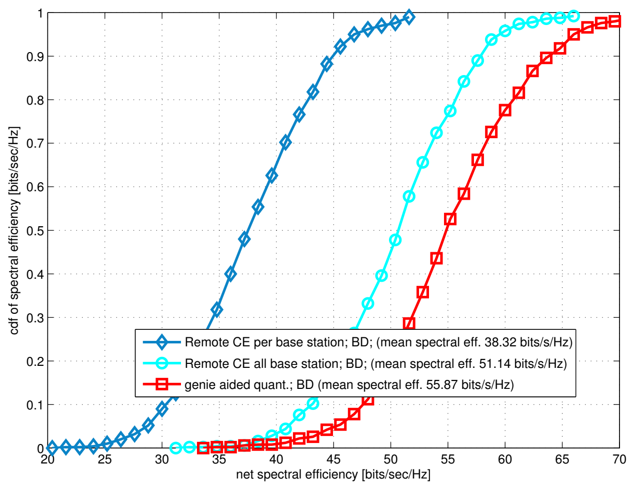

Fig. 2. Spectral efficiency over feedback bits. Comparing remote channel estimation per base station with remote channel estimation over multiple base stations. Simulation setup given in Table I.

IQ channel quantization with ZFBF: Is outperformed by rate approximation for low feedback rates (i.e. below 0.4 kbits per feedback message) and outperforms coherent transmission for feedback rates below 1.5 kbits per feedback message (except genie aided quantization).

Another advantage of the proposed remote channel estimation is that the channel can be quantized over multiple base stations. This approach inherently adapts the number of feedback bits between different base stations such that the channels to "stronger" base stations are more accurately quantized than channels to "weaker" base stations. In Figure 2 we compare the performance of the remote channel estimation scheme if the channels to all $T = 3$ base stations are quantized all at once or separately per base station. We assume the same feedback rate of approx. 2.5 kbits per feedback message. The all base stations at once channel quantization approach clearly outperforms the approach where the channel to each base station is quantized separately with B/T bits.

V. Conclusions

We presented a remote channel estimation feedback scheme that moves the channel estimation process to the transmitter side. The proposed feedback was shown to have many advantages over conventional schemes. The complexity at the receiver site can be significantly reduced. Using the proposed scheme accurate CSIT is available per subcarrier. The number of feedback bits used to quantize the channels from different transmitters is inherently adapted, such that strong transmitters get potentially more feedback bits than weak transmitters.

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