Robust Path Detection for the LTE Downlink based on Compressed Sensing

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***Abstract*— We will present an overview and simulation results of channel estimation exploiting sparsity for the concrete parameters of the LTE system. Recent progress in compressed sensing has shown that $\mathcal{O}(s\log(p)^4)$ randomly placed pilots are sufficient to recover a s–sparse impulse response of length p by solving a convex relaxation of the combinatorial sparsity problem. The pilot configuration in LTE is already standardized and a support for random pilot placement is not included. But we show that also in this fixed and deterministic setting substantial gains have to be expected from non–conventional channel estimation that make explicit use of the sparse nature of mobile communication channels.**

I. Introduction

It is theoretically well–known that coherent transmission increases the overall performance of nowadays communication systems like *Long Term Evolution* (LTE). Coherent transmission strategies can be enabled for example by using pilot–based channel estimation. But optimal estimation strategies depend much on the used model for the mobile communication channel and how well these assumptions approximate the nature of the true channel. It can be observed that the impulse response of multipath channels present at moderate velocities concentrates in several clusters, so that the sampled response is *sparse*, i.e. has only a few non–zero samples. A reliable channel estimation stage should therefore take the sparsity property into account. In a multiuser downlink based on *Orthogonal Frequency Division Multiplexing* (OFDM) like LTE, common pilots are placed uniformly in the allowed frequency band to allow the users to estimate their channels. The pilot spacing is determined by the Nyquist criterion, i.e. motivated from perfect reconstruction of *any* impulse response supported on a certain interval. Also, for more general pilot placements it is known how to setup ℓ_2–estimators (like LMMSE and LS). However, all these approaches do not make explicit use of sparsity and a relevant amount of resources therefore have to be spent for the pilots.

Recent progress in compressed sensing has shown that a sparse impulse response can be recovered from much less *randomly placement* pilots [1]. Channel estimation in this direction (random measurements) is already discussed for example in [2], [3]. However, it is still unclear how the theoretical results apply on such a concrete application like LTE, in particular with its deterministic pilot placement. In this paper we will investigate such a new approach and compare its performance with conventional ℓ_2–estimators. We will discuss the impact and relevance for the LTE system.

II. Background on Channel Estimation

It is established for example in [4] that pilot–based estimation of the time–invariant channel in an OFDM system can be formulated as:

$$g = W_T h_T + z$$

The vector h_T denotes here the $|T|$ coefficients of the channel impulse response on a certain a–priori given sampling set $T = \{\tau_n\}$. The vector g contains noisy $|F|$ observations on a particular subset $F = \{f_p\}$ of the in total K discrete frequencies. The vector z denotes the additive white Gaussian noise of power σ^2. Hence, W_T is the submatrix of the DFT–matrix (transmitted pilot values are assumed to be one) with rows in F and columns in T:

$$(W_T)_{pn} = e^{i2\pi\tau_n f_p/K} \tag{1}$$

We assume that the length CP of cyclic prefix is designed appropriately to avoid intersymbol interference. In any initial estimation stage the positions of the $s' \ll$ CP resolvable paths are unknown, in general. The performance of final channel estimation stage therefore depends crucially on how reliable the dominating paths and its positions can be determined.

III. Standard LMMSE and LS Estimates

Let $T \subseteq [0, \dots, N-1]$ be an arbitrary subset of possible tap positions and $R_T \in \mathbb{C}^{|T|\times|T|}$ be the correlation matrix of the random vector of channel coefficients h_T. Usually one assumes that $R_T = \text{diag}(p_h)$ where the vector p_h contains the T–samples of the channels power delay profile. We abbreviate then with the matrix Q_T the LMMSE estimator minimizing the total mean squared error (MSE). For our assumption on the noise z we have the standard forms:

$$\begin{aligned} Q_T &= R_T W_T^* \left(W_T R_T W_T^* + \sigma^2 \right)^{-1} \\ &= \left(W_T W_T^* + \sigma^2 R_T^{-1} \right)^{-1} W_T^* \end{aligned} \tag{2}$$

However, in practice also R_T is unknown and has to be estimated as well. From (2) follows that if "increasing" R_T without bound the estimator Q_T becomes the LS (also called Gauss–Markov) estimate having the potential advantage of being independent of the *non–zero values* of the power delay profile (and σ^2). The *conventional LMMSE estimate* is then $Q_C \cdot g$ with $C := [0, \dots, \text{CP} - 1]$ and R_T being a scaled identity. Similarly, the *oracle LMMSE estimate* is $Q_S \cdot g$ where the set S of all s' instantaneous positions and the corresponding power delay profile is already known without error.

IV. Compressed–Sensing based Estimates

Most of the results in sparse estimation are formulated in the real domain. Thus, let us set $s = 2s'$, $p = 2p'$ and $n = 2|F|$ and define as in [2] the *real* matrix:

$$\Phi = \frac{1}{\sqrt{n'}} \begin{pmatrix} \text{Re}\{W_T\} & -\text{Im}\{W_T\} \\ \text{Im}\{W_T\} & \text{Re}\{W_T\} \end{pmatrix} \in \mathbb{R}^{n\times p} \tag{3}$$

The real vector $x = [\text{Re}\{h_T\}, \text{Im}\{h_T\}] \in \mathbb{R}^p$ contains T–samples of the impulse response h and $y = [\text{Re}\{g\}, \text{Im}\{g\}] \in \mathbb{R}^n$ are the observations. We use here the common terminology: $\|x\|_0 := |\{k \,|\, x_k \neq 0\}|$ denotes the cardinality of the support of $x \in \mathbb{R}^p$ and x is said to be s–sparse if $\|x\|_0 \leq s \ll p$.

A. Sparse Estimation

Estimation methods for sparse vectors will depend much on how well Φ behaves as an isometry on small coordinate subsets which can measured with the *restricted isometry property* (RIP) [5]. The matrix Φ is said be s–RIP if there exists $\delta_s < 1$ such that for all $c \in \mathbb{R}^{|T'|}$ it holds:

$$(1-\delta_s)\|c\|_2^2 \leq \|\Phi_{T'} c\|_2^2 \leq (1+\delta_s)\|c\|_2^2$$

for all subsets $T' \subseteq [1 \dots p]$ of cardinality $|T'| \leq s$. In other words: $(1 \pm \delta_s)$ are uniform bounds for the spectrum of Gram matrices $\{\Phi_{T'}^* \Phi_{T'}\}$. It is clear that computing δ_s can be quite complex. Another more accessible parameter of Φ is the (mutual) *coherence*:

$$\mu = \max_{m \neq n} |\langle \phi_m, \phi_n \rangle| \tag{4}$$

where $\{\phi_m\}$ are the rows of Φ. Coherence is indirectly linked with sparsity, that is $\delta_s \leq (s-1)\mu$ which follows from Gershgorin's theorem. Let $B_q(y, \epsilon)$ be the ℓ_q–ball in $\mathbb{R}^p$ of radius ϵ around y, i.e. $B_q(y, \epsilon) := \{c \in \mathbb{R}^p \,|\, \|c - y\|_q \leq \epsilon\}$. For white Gaussian noise it follows that $\Phi \cdot x \in B_2(y, \epsilon)$ with high probability for appropriate ϵ. Now we select the sparsest candidate as:

$$\hat{x}_0(\epsilon) = \min_{\Phi \cdot c \in B_2(y,\epsilon)} \|c\|_0 \tag{5}$$

which still matches the observations y within ϵ. The minimum is unique for $\epsilon = 0$ if $\delta_{2s} < 1$ and solving (5) exactly recovers in this case any s–sparse x if $\delta_{2s} + \delta_{3s} < 1$ [5], i.e. $\hat{x}_0(0) = x$. However, problem (5) is NP–complete in general, i.e. $\binom{p}{s}$ combinations have to be checked.

B. Convex Relaxation

A convex relaxation of problem (5) is the quadratically constrained linear program:

$$\hat{x}_1(\epsilon) = \arg \min_{\Phi \cdot c \in B_2(y,\epsilon)} \|c\|_1 \tag{6}$$

extending the called *basis pursuit* estimate for imperfect (noisy) measurements [6]. The Lagrangian of this program leads to the ℓ_1–regularized quadratic program [6] known as the *basis pursuit denoising* estimate:

$$\hat{x}_{12}(\tau) = \arg \min_c \frac{1}{2}\|\Phi \cdot c - y\|_2^2 + \tau \cdot \|c\|_1 \tag{7}$$

where τ is some regularization parameter. From convex analysis follows that the problems are equivalent in a sense, that $\hat{x}_1(\epsilon) = \hat{x}_{12}(\tau)$ for appropriate ϵ and τ. Note that, τ in (7) is an ℓ_1–regularization parameter sensitive to sparsity whereby (2) is a weighted variant of Tikhonov regularization with σ^2 as ℓ_2-regularization parameter being not sensitive to sparsity.

Reconstruction Performance: In [5] it was shown that in the noiseless case $\delta_s + \delta_{2s} + \delta_{3s} < 1$ is a sufficient condition for $\hat{x}_0(0) = \hat{x}_1(0) = x$. It has been shown further in [7] that the MSE of the program (6) can upperbounded as:

$$\|\hat{x}_1(\epsilon) - x\|_2 \leq C_s \cdot \epsilon \tag{8}$$

for all s–sparse x if $\delta_{3s} + 3\delta_{4s} < 2$. Program (7) and the relation to coherence μ was intensively studied in [8] and in particular it was found that for $s \leq 1/(3\mu)$ and $\Phi \cdot x \in B_2(y, \epsilon)$ the solution $\hat{x}_{12}(2\epsilon)$ of (7) is unique and in the support of x with an uniform reconstruction error per component below $(3 + \sqrt{3/2}) \cdot \epsilon$ giving also a bound for the MSE.

Results for Fourier Measurements: Most of the results for Fourier measurements are obtained from concentration principles. For example, it is known that a given s–sparse x can be reconstructed with overwhelming probability from $\mathcal{O}(s \log(p))$ noiseless pilots at random positions [9]. Unfortunately this statement is not uniform in any s–sparse x. But it is conjectured that this scaling is uniform in general. The up to now best uniform relation between the isometry constant δ_s and the corresponding number n of measurements is due to Rudelson and Vershynin [1] stating that for $\mathcal{O}(s \log(p) \log^2(s) \log(s \log p))$ s–sparse signals are recovered with high probability. This has been formulated in a more concrete form by Rauhut [10] for the MSE of algorithm (6).

Algorithms: The problem (7) can be casted as a quadratic program on a cone by splitting the vector $c = c^+ - c^-$ into its positive and negative parts $z := [c^+, c^-] \geq 0$. Then problem (7) is equivalent to $\min_{z \geq 0} \langle d, z \rangle + \langle z, \Psi z \rangle / 2$ where $\Psi = (\Phi^* \Phi) \otimes \left(\begin{smallmatrix} +1 & -1 \\ -1 & +1 \end{smallmatrix}\right)$ and $d = [\tau - \Phi^T y, \tau + \Phi^T y]$ which can be solved for example with the GPSR algorithm presented in [11]. A matlab code is provided by the authors on `http://www.lx.it.pt/~mtf/GPSR`.

V. Results on Sparse Estimation for LTE

The upcoming LTE standard provides a platform for a cellular multiuser system with support for multiple antennas at the basestations and the terminal. Resources can be assigned in frequency, time and spatial dimensions. In the downlink the system uses OFDM modulation and common pilots are transmitted from each basestation antenna on certain frequencies separately.

A. Pilot Configuration and Sparsity Properties

For simplicity for consider here only the 5MHz configuration (sampling frequency is 7.68MHz, $K = 512$ and CP $= 40$). The useful frequency band consists of about 300 subcarrier and is subdivided into groups of 12 consecutive subcarriers for frequency–selective scheduling. A transmission block (TTI) of 1ms contains 14 OFDM symbols. The resources in one TTI assigned to a single user carry a single turbo coded data block. Here we consider a frequency resource of 3×12 subcarriers. The pilots are scattered in time and in the allowed frequency band with patterns which ensures orthogonality between different transmit antennas within one cell such that the setup explained in Section II is applicable. The standard pilot spacing is 6 subcarriers, hence we have complex measurements matrices W_T defined in (1) with $|F| = 50$ and $|T| = 40$.

RIP and Coherence: The coherence μ in (4) can be computed directly and for pilot spacings $[3, 6, 12, 24]$ one obtains always $\mu > 0.5$, i.e. the sparsity condition $s \leq 1/(3\mu)$ is useless for these parameters. There exists several (non–trivial) methods of verifying the RIP properties which is important for large scale problems. However, in the present application δ_s (or at least lower bounds) can simulated directly. Unfortunately, we observe:

sparsity s	bound	1	2	6
$\delta_{2s} + \delta_{3s}$	< 1	≈ 1.3	≈ 1.6	≈ 1.7
$\delta_s + \delta_{2s} + \delta_{3s}$	< 1	≈ 1.3	≈ 2.1	≈ 2.5
$\delta_{3s} + 3\delta_{4s}$	< 2	≈ 3.0	≈ 3.3	≈ 3.4

that the sufficient sparsity condition are not fulfilled for the 5MHz configuration (for the 10MHz as well). But it is also known in the community that sparse estimation can work much beyond this regime.

Channel Estimators: We will study the following combined channel estimation techniques (GPSR&LMMSE): For path detection we solve (7) with the GPSR algorithm (with Barzilai–Borwein projection) to estimate the support set T. For this particular T we apply a final LMMSE stage (2). We compare this approach to the conventional and oracle LMMSE methods explained in Section III. For the link-level throughput evaluation we compare this also to the case with ideal channel state information.

B. MSE Simulations

A direct performance measure is the MSE of the channel coefficients per data subcarrier. We observe from Figure 1 a difference of about 6dB between the conventional LMMSE estimate having no knowledge of the true path positions and the oracle LMMSE. Running GPSR with a certain fixed τ gives a slight improved performance at low SNR but at 15dB it meets already the conventional LMMSE estimation and achieves a floor. Different τ's will give a different behavior of the performance and the different crossing points. It is not a–priori clear how

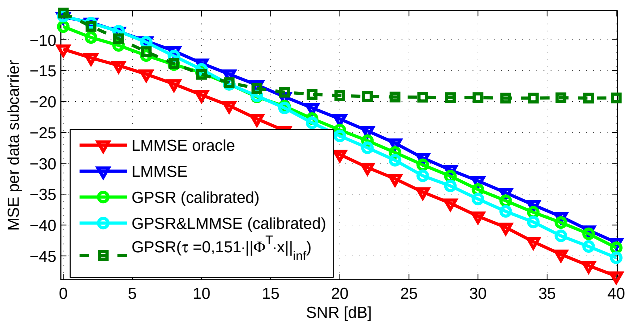

Fig. 1. The MSE per data subcarrier of different channel estimators is shown for a 6-tap channel with uniformly distributed path positions $\tau \in [0, \mathrm{CP}-1]$ and Rayleigh distributed values with power law $p_h(\tau) = e^{-\tau/100}$.

τ should be adapted optimally (except the scaling with σ). As indicated also in [11] a reasonable choice (also for convergence behavior) is to write $\tau = c(\sigma) \cdot \|\Phi^T y\|_\infty$ and use $c(\sigma)$ as a calibration for the channel estimation algorithm. We have simply measured this dependency once. We observed essentially no practical difference in the calibration between different statistical channel models. While this approach achieves now at all SNR points a better performance as the conventional LMMSE the gain at high SNR (40dB) becomes negligible. A reason could be that from a certain SNR point on the path positions are correctly identified in most cases but the non–zero channel coefficients are not recovered reasonable in the ℓ_2–sense. In this case we have to expect a further gain with "GPSR&LMMSE" approach. As can be seen in Figure 1 that this is indeed the case. However, the success rate of the path identification decreases at lower SNR such that the final LMMSE stage will further degrade the performance.

One might expect that such path detection methods will work reliable for really sparse channels but break down quite fast in the non–sparse setting. However, from [7] is known that the solution $\hat{x}_1(\epsilon)$ of (8) is in the non-sparse case a robust recovery of the s entries of x having the largest magnitude. This can also be observed from Figure 2 where the MSE is shown at SNR 20dB for an increasing

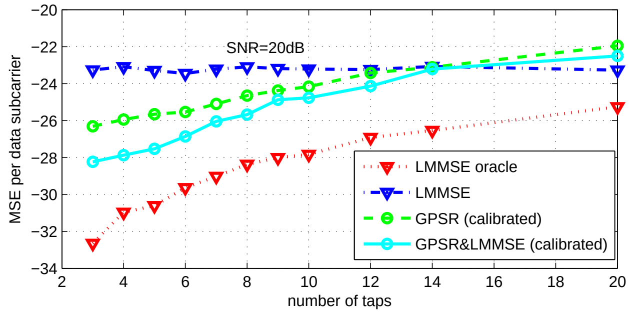

Fig. 2. Stability with increasing number of taps at SNR of 20dB. Further parameters are as in Figure 1.

number of paths. For this particular setup we see that below 14 channel taps sparse estimation outperforms the conventional LMMSE estimate. For more than 14 channel taps sparse estimation starts to fail in identifying the correct tap positions.

C. *Linklevel Throughput Evaluation*

We have used the SCME channel simulator [12] which generates a "urban macro"–like time–varying channel having 6 paths with positions (almost) within the cyclic prefix. We use a basestation with 4 transmit antennas in $\lambda/2$ configuration and mobiles having a single receive antenna (i.e. a MISO setup). The mobiles are at a distance of 200m to the basestation and uniformly distributed in a 120° sector. The carrier frequency is 2 GHz.

Single User Performance: We investigated here an approach where a fixed transmit beamforming codebook is used. After estimating the channel from the common pilots each mobile reports its "best beam" for the 3×12 subcarriers as *channel direction information* (CDI) and a corresponding *channel quality information* (CQI). We use a tapered codebook with 8 elements (CDI is 3bits) as proposed in [13]. In Figure 3 we see the envelope throughput over 3 different modulation and coding schemes (QPSK,16QAM, 64QAM with turbo coding). We can observe that the combined GPSR–LMMSE channel estimation achieves the same throughput as the oracle estimator.

Multi User Performance: A specific feature of LTE is the multiuser MIMO operation mode. In our example we have 8 users in the sector and the basestation pairs to the strongest (in CQI) user the second strong user outside a beam distance of 2 for simultaneous transmission [13] (with equal power). In Figure 4 it shown that again the same sum throughput as with the oracle estimator is achieved with the combined GPSR–LMMSE approach.

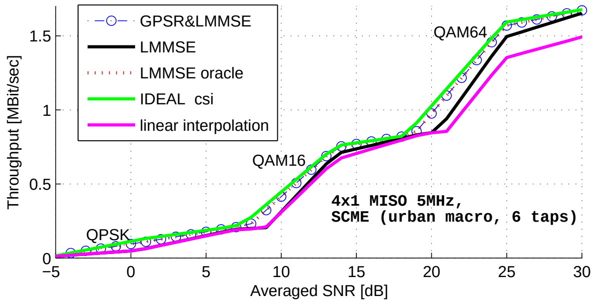

Fig. 3. Single-User MISO envelope throughput for the approach in [13]. The channel estimation is used for feedback and data equalization. Furthermore the performance of simple linear interpolation between pilot tones is included.

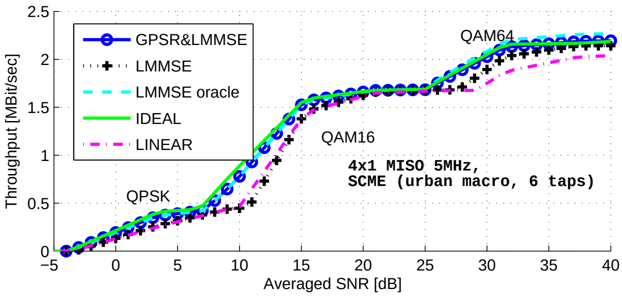

Fig. 4. Multi-User MISO envelope sum throughput. In contrast to Figure 3 the channel estimation is used here also for selecting a *user pair* from 8 users in the sector.

Varying the Pilot Spacing: A potential advantage of using sparse estimation could be the reduced number of necessary pilot tones. In Figure 5 we see that for the proposed combined approach which exploits sparsity much less pilots are necessary.

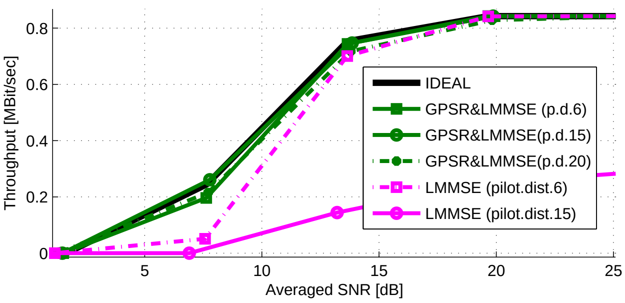

Fig. 5. Single-User MISO throughput (16QAM only) with different pilot spacings. Further settings are as in Figure 3. We observe that GPSR&LMMSE is almost stable when decreasing the number of pilot tones whereby conventional LMMSE fails.

VI. Conclusions

We have evaluated methods of channel estimation exploiting the sparsity of mobile communication channels. We have observed that current theoretical bounds are still too weak and not applicable to the usual LTE setup. However, under moderate conditions as present in typical LTE environments, we have obtained substantial gains with channel estimation based on sparsity over conventional LMMSE methods. Furthermore, the proposed channel estimator keeps almost stable when reducing the pilot overhead. This might be an important detail for pilot assignment and sequence planning in the multicell context.

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