Transformation of Petri Nets with Individual Tokens

Tony Modica[1], Karsten Gabriel[2], Kathrin Hoffmann[3]

[1] modica@cs.tu-berlin.de –
Integrated Graduate Program Human-Centric Communication (H-C3), TU Berlin
[2] kgabriel@cs.tu-berlin.de –
Fraunhofer Institute for Open Communication Systems (FOKUS), Berlin
[3] kathrin.hoffmann@haw-hamburg.de –
Hochschule für Angewandte Wissenschaften, Hamburg

Petri nets are one of the main formalisms to describe and analyze concurrent processes. They have been a promising candidate not only for extensions but also for integrations with different formal techniques to capture more complex aspects.

Petri net transformation based on graph transformation [4] has been successfully applied to adaptive workflows in [7,1] to change the net structure. Moreover, rule-based transformation is also available for Petri systems, i.e. Petri nets with an initial marking, which is especially useful for manipulation of processes at runtime.

Place/transition (P/T) nets have been shown in [3,4] to form a weak adhesive HLR category with the class $\mathcal{M}$ of all injective net morphisms. This allows us to apply all the results for adhesive HLR systems concerning the local Church-Rosser property, parallelism, and concurrency of transformations as shown in [3] also for P/T net transformation systems. In this paper, we use the short notion "$\mathcal{M}$-adhesive category" for "weak adhesive HLR category". The concept of Petri systems leads to a category **PTSys** with morphisms allowing to increase the number of tokens on corresponding places. Unfortunately, $(\mathbf{PTSys}, \mathcal{M}_{inj})$ with the class $\mathcal{M}_{inj}$ of all injective morphisms is not $\mathcal{M}$-adhesive in contrast to $(\mathbf{PTSys}, \mathcal{M}_{strict})$, where $\mathcal{M}_{strict}$ is the class of injective morphisms where the number of tokens on corresponding places is equal. This means that we can not formulate rules in $(\mathbf{PTSys}, \mathcal{M}_{strict})$ that change markings, which is an unpleasant restriction for the usability of the transformation approach. Especially the firing of a transition cannot be simulated in a natural way by the application of a corresponding "transition rule".

To overcome this restriction we present a new Petri net formalism, called "place/transition nets with individual tokens" or short PTI nets, together with a rule-based transformation approach that is almost equivalent to the "collective" one in [3,4]. The pivotal difference concerns the representation of net markings: for the new individual approach, we propose to define a net's marking as a set of individuals instead of a (collective) sum of a monoid. Nevertheless, nets with individual tokens still follow the concept "Petri nets are monoids" from [8].

The approach of individual tokens is closely related to the "individual token interpretation" of firing steps in [6,2] that considers not only the number and

value of tokens but also the history, i.e. the origin transition, of tokens in firing sequences. In contrast, we integrate individual tokens in the syntactical definition instead of the semantical one.

A main result is that (**PTINets**, $\mathcal{M}$) with the class $\mathcal{M}$ of all injective morphisms forms an $\mathcal{M}$-adhesive category with all its nice properties for which we can formulate rules that change the marking of a net independently from changes of the net's structure.

With an explicit construction of initial pushouts we give necessary and sufficient conditions for the applicability of transformation rules in these categories, known as gluing condition in the literature. As a second main result we show the equivalence of firing steps with corresponding transition firing rules. Hence the result of independent firing and transformation steps [5] is achieved in a more general and elegant way by using the well-known results of independent transformation steps [3].

We motivate and demonstrate our new approach with a simple example on modeling data recovery in a distributed communication system. Moreover, we give an outlook on algebraic high-level nets with individual tokens to model especially highly dynamic structures and complex behavior in the area of communication platforms in an adequate way.

References

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