Robotics
Exercise 8

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1 Bayes Basics

a) Box 1 contains 8 apples and 4 oranges. Box 2 contains 10 apples and 2 oranges. Boxes are chosen with equal probability. What is the probability of choosing an apple? If an apple is chosen, what is the probability that it came from box 1?

b) The Monty Hall Problem: I have three boxes. In one I put a prize, and two are empty. I then mix up the boxes. You want to pick the box with the prize in it. You choose one box. I then open *another* one of the two remaining boxes and show that it is empty. I then give you the chance to change your choice of boxes—should you do so?

c) Given a joint probability $P(X, Y)$ over 2 binary random variables as the table

	Y=0	Y=1
X=0	.06	.24
X=1	.14	.56

What are $P(X)$ and $P(Y)$? Are X and Y conditionally independent?

2 Gaussians

On slide 06:17 there is the definition of a multivariate (n-dim) Gaussian distribution. Proof the following using only the definition. (You may ignore terms independent of x.)

a) Proof that:

$\mathcal{N}(x|a, A) = \mathcal{N}(a|x, A)$

$\mathcal{N}(x \,|\, a, A) = |F| \; \mathcal{N}(Fx \,|\, Fa, \; FAF^\top)$

$\mathcal{N}(Fx + f \,|\, a, A) = \frac{1}{|F|} \; \mathcal{N}(x \,|\; F^{-1}(a - f), \; F^{-1}AF^{-\top})$

b) Multiplying two Gaussians is essential in many algorithms (typically, a prior and a likelihood, to get a posterior). Prove the general rule

$$\begin{aligned} &\mathcal{N}(x \,|\, a, A) \; \mathcal{N}(x \,|\, b, B) \\ &\propto \mathcal{N}(x \,|\, (A^{-1} + B^{-1})^{-1}(A^{-1}a + B^{-1}b), (A^{-1} + B^{-1})^{-1}) \;, \end{aligned} \tag{1}$$

where the proportionality $\propto$ allows you to drop all terms independent of x.

Note: The so-called canonical form of a Gaussian is defined as $\mathcal{N}[x \,|\, \bar{a}, \bar{A}] = \mathcal{N}(x \,|\, \bar{A}^{-1}\bar{a}, \bar{A}^{1})$; in this convention the product reads much nicher: $\mathcal{N}[x \,|\, \bar{a}, \bar{A}] \; \mathcal{N}[x \,|\, \bar{b}, \bar{B}] \propto \mathcal{N}[x \,|\, \bar{a} + \bar{b}, \bar{A} + \bar{B}]$.

c) The "forward propagation" of a Gaussian belief $\mathcal{N}(y \,|\, b, B)$ along a stochastic linear dynamics $\mathcal{N}(x \,|\, a + Fy, A)$ is given as

$$\int_y \mathcal{N}(x \,|\, a + Fy, A) \; \mathcal{N}(y \,|\, b, B) \; dy = \mathcal{N}(x \,|\, a + Fb, A + FBF^\top) \tag{2}$$

Prove the equation.