Introduction to Optimization
Exercise 4

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1 Squared Panalties & Log Barriers

In a previous exercise we defined the "hole function" $f^c_{\text{hole}}(x)$, where we now assume a conditioning $c = 4$. Consider the optimization problem

$$\min_x f^c_{\text{hole}}(x) \quad \text{s.t.} \quad g(x) \le 0 \tag{1}$$

$$g(x) = \begin{pmatrix} x^\top x - 1 \\ x_n + 1/c \end{pmatrix} \tag{2}$$

a) First, assume $n = 2$ ($x \in \mathbb{R}^2$ is 2-dimensional), $c = 4$, and draw on paper what the problem looks like and where you expect the optimum.

b) Implement the Squared Penalty Method. (In the inner loop you may choose any method, including simple gradient methods.) Choose as a start point $x = (\frac{1}{2}, \frac{1}{2})$. Plot its optimization path and report on the number of total function/gradient evaluations needed.

c) Test the scaling of the method for $n = 10$ dimensions.

d) Implement the Log Barrier Method and test as in b) and c). Compare the function/gradient evaluations needed.

2 Lagrangian and dual function

(Taken roughly from 'Convex Optimization', Ex. 5.1)

A simple example. Consider the optimization problem

$$\min x^2 + 1 \quad \text{s.t.} \quad (x-2)(x-4) \le 0$$

with variable $x \in \mathbb{R}$.

a) Derive the optimal solution x^* and the optimal value $p^* = f(x^*)$ by hand.

b) Write down the Lagrangian $L(x, \lambda)$. Plot (using gnuplot or so) $L(x, \lambda)$ over x for various values of $\lambda \ge 0$. Verify the lower bound property $\min_x L(x, \lambda) \le p^*$, where p^* is the optimum value of the primal problem.

c) Derive the dual function $l(\lambda) = \min_x L(x, \lambda)$ and plot it (for $\lambda \ge 0$). Derive the dual optimal solution $\lambda^* = \operatorname{argmax}_\lambda l(\lambda)$. Is $\max_l l(\lambda) = p^*$ (strong duality)?