

## Uncertainty Measures for Sensor Management in a Survivability Application

Tina Erlandsson (Saab AB)  
Lars Niklasson (University of Skövde)

tina.erlandsson@saabgroup.com  
lars.niklasson@his.se

**Abstract:** When flying a mission, a fighter pilot is exposed to the risk of being hit by enemy fire. A tactical support system can aid the pilot by calculating the survivability of a given route, which is the probability that the fighter pilot can fly the route without being hit. The survivability estimate will be uncertain due to uncertainty in the information about threats in the area. In this paper, we investigate the uncertainty in the estimate of the survivability and compare two different measures of uncertainty; standard deviation and entropy. Furthermore, we discuss how these measures can be used for sensor management and discuss a few issues that need to be addressed in the design of a sensor management system in a fighter aircraft.

**Keywords:** Survivability, uncertainty, sensor management, fighter aircraft

### 1 Introduction

A fighter pilot simultaneously has to fly the aircraft, focus on the mission and avoid enemy fire. Schulte's goal model [Sch01] describes this as three objectives; flight safety, combat survival and mission accomplishment: These objectives often conflict each other during flight, for instance when the pilot has to fly within a hostile area in order to accomplish the mission. Inside hostile areas there is a risk that the aircraft will be hit by enemy fire. A tactical support system that aid the pilot with analyzing this risk can be designed based on the survivability model proposed in [ENNW11]. The survivability describes the probability that the pilot can fly the mission route without being hit by enemy fire.

The survivability calculations will be based on estimates about the threats in the area, such as the number of threats, their positions and capabilities. The estimates are typically based on information from intelligence sources that has been preloaded into the aircraft before the mission and data from the onboard sensors and possibly also sensor data from other aircraft within the team. Unfortunately, this information will in practice be uncertain and this will induce uncertainty in the survivability calculations and the risk analysis. To decrease this uncertainty it is desirable to gather more information so that more precise estimates of the threats can be achieved. This can be accomplished with a sensor management system that optimizes the use of the sensors.

Sensor management is in [XS02] described as “*a system or process that seeks to manage or coordinate the usage of a suite of sensors or measurement devices in a dynamic uncertain environment, to improve the performance of data fusion and ultimately that of perception*”. It has been argued that a sensor management system can enhance the attack capabilities of an attack aircraft by for instance decreasing pilot workload, increasing pilot situation awareness and reduce redundant sensor usage [BRM88]. Sensor management includes a number of issues that need to be addressed, such as architecture, sensor task scheduling, sensor placement etc. Surveys of the problems and approaches for sensor management described in the literature is given in [XS02, NN00].

According to [NN00], there is an increased awareness of the importance of sensor management research to go beyond scheduling and also study how resource allocation can be optimized in accordance to the specific mission goals. As argued in [Mal95], approaches that focus on gaining as much information as possible at each particular time can be myopic. Instead the value of the information should be taken into account. This requires a performance index that values the possible information that can be gathered. The aim of the sensor management is then to optimize the sensor usage against this performance index, which can include probability of target detection, track/identification accuracy, probability of loose-of-track, probability of survival, probability of target kill etc [XS02]. Based on this argument, the focus in this paper is to study how the uncertainty in threat position influences the uncertainty in survivability. Two uncertainty measures are studied and the idea is that one of these uncertainty measures could be used as performance index for a sensor management system. Furthermore, we discuss a few issues that need to be addressed in the design of a sensor management system in a fighter aircraft.

## 2 Survivability Model

The survivability model proposed in [ENNW11] calculates the probability that the aircraft can fly the mission route without being hit, see Figure 1. It is based on stochastic processes and survival analysis, further described in for instance [Blo84, YG05] and is inspired by work within maintenance engineering, where similar models are used for calculating the remaining life time of a component [BDDR<sup>+</sup>09].

The survival function  $R(t)$  describes the probability that the aircraft has not been hit up to time  $t$ . Thus for every time instance of the route, it gives the probability that the aircraft reaches the point without being hit. The survival function is expressed as:

$$R(t) = Pr(\text{not hit at time } t) = \exp\left(-\int_0^t \lambda(u)du\right) \quad (1)$$

$\lambda(t)$  is known as the intensity or hit rate <sup>1</sup>. Furthermore

$$\lambda(t) = \lim_{\Delta t \rightarrow 0} \frac{Pr(T_{hit} < t + \Delta t | T_{hit} > t)}{\Delta t} \quad (2)$$

Thus  $\lambda(t)$  can be interpreted as the rate of the conditional probability that the aircraft will be hit, given that it has not already been hit. The survivability model requires that  $\lambda$  can be described for all time instances on the route. In order to do this, we need to model the threats that are positioned along the route and connect them with  $\lambda$ . In this paper, we use the simple model proposed in [ENNW11] where a threat is described with a stationary position and a threat area, see Figure 1. The intensity  $\lambda$  have a constant value within the threat area, and is zero outside. When  $\lambda$  is constant, the expected time before hit is given by  $\frac{1}{\lambda}$ . Thus the intensity for a threat is described as inverse of the mean time before hit for that threat. Needless to say, this is a naive simplification, but it here serves our purpose to investigate different uncertainty measures. More sophisticated threat descriptions are discussed in [ENNW11].

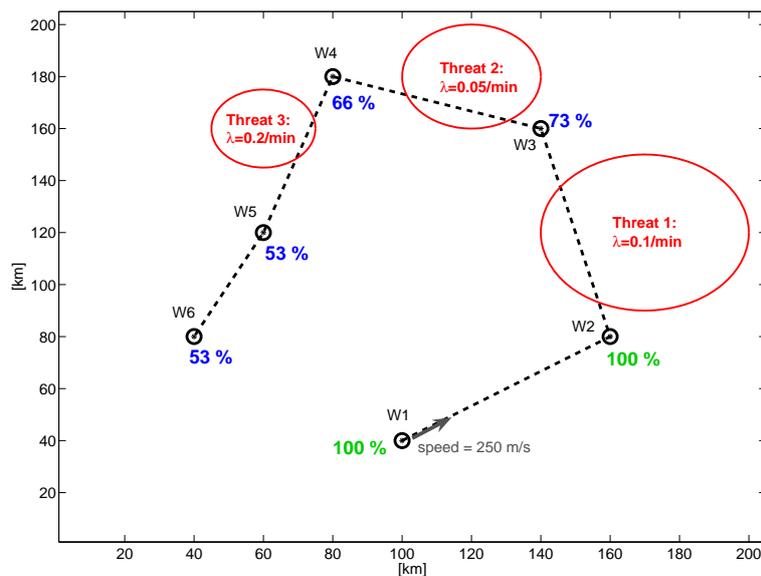


Figure 1: The figure shows the route with waypoints (W1-W6), that the pilot intends to fly. There are three threats located along the route with different intensity  $\lambda$ . The survivability has been calculated for each waypoint, i.e. the probability that the aircraft will not be hit before it reaches the point.

<sup>1</sup>In maintenance engineering the survival function is called reliability function and describes the probability that no component failure has occurred at time  $t$ . Furthermore  $\lambda$  is called failure rate or the hazard function cf. [BDDR<sup>+</sup>09].

### 3 Uncertainty in Survivability

Uncertainty can be classified as aleatory/irreducible uncertainty or epistemic/reducible uncertainty [OHJ<sup>+</sup>04]. The survivability model is a stochastic model, meaning that it can not be known for certain whether a route is safe or not; the model can only describe the probability that the aircraft will be able to fly the route without being hit. The uncertainty in the model is thus aleatory or irreducible uncertainty, since it is not possible to reduce the uncertainty. In order to increase the survivability, the pilot can for instance fly another route or apply radar jamming to deceive the threat's tracking system.

Unfortunately, the calculations of the survivability will in practice also be uncertain due to the uncertainty in the input data, which will consist of intelligence information and sensor data. The intelligence information typically describes which types of threats that can be expected in the area and information about them, such as type of weapon, fire range, and hit rate, which is used to describe  $\lambda$ . There might also be intelligence reports indicating the location of known threats. The sensors on the aircraft can provide data about the position of the threat and also give an estimate of the type of threat. The sensor data is uncertain due to sensor noise and limited detection distance. The intelligence information is also uncertain since information about weapon capacity is typically held secret by the enemy and must be estimated by military experts. This uncertainty is epistemic since it is possible to reduce it if more information is received.

#### 3.1 Uncertainty Measures

In order to investigate the epistemic uncertainty, we have performed Monte Carlo simulations with the scenario depicted in Figure 1, Section 2. The positions of the threats have been drawn from a Gaussian distribution with expected value of position as in the scenario and with different standard deviations and 10000 simulations for each case. The simulation results have been summarized into histograms. Even though the histograms contain a lot of information, they may be cumbersome to work with. Especially in the case of sensor management where the aim is to automatically minimize the uncertainty, it would be desirable to have a scalar measure of the uncertainty. This section describes two possible uncertainty measures and how they can be estimated from the simulation results.

##### Standard deviation

The standard deviation is defined as  $\sigma = \sqrt{E[(X - E(X))^2]}$  and indicates the expected difference between an observation of a random variable  $X$  and its expected value [YG05]. It is estimated from the simulation results with

$$\bar{s} = \left( \frac{1}{N-1} \sum_{i=1}^N (x_i - \bar{x})^2 \right)^{\frac{1}{2}} \quad \text{where } \bar{x} = \frac{1}{N} \sum_{i=1}^N x_i. \quad (3)$$

$N$  is the number of simulations and  $x_i$  is the survivability in simulation  $i$ .

### Shannon's Entropy

In information theory, Shannon's entropy of a random process is defined as  $H(X) = -\sum_i p(x_i) \log(p(x_i))$ . This measure has been suggested for sensor management, where it is used for calculating the information gain with an observation by comparing the entropy of the distribution before the observation with the estimated distribution after the observation [XS02]. To estimate the entropy from a histogram of the simulation result, we use:

$$\bar{H} = -\frac{1}{\sum n_k} \sum_{n_k \neq 0} n_k \ln(n_k) \quad (4)$$

where  $n_k$  is the number of simulations in the  $k$ :th bar in the histogram and  $\ln$  denotes the natural logarithm.

### 3.2 Simulation Results

Figure 2 depicts a scaled histogram of the simulation result of survivability for the last waypoint (W6), i.e. the survivability for the entire route.

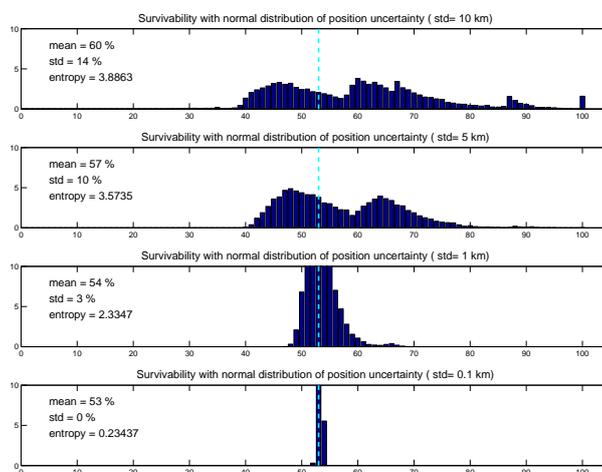


Figure 2: Histogram of the results from Monte Carlo simulations of the survivability at the last waypoint (W6) with the position drawn from a two-dimensional normal distribution with expected value as the position in the scenario in Figure 1 and different standard deviations. The histogram has been scaled so that 1 on the y-axis represent 100 simulations. The mean survivability and estimated standard deviation and entropy are also displayed.

The scaled histogram can be seen as an estimate of the density distribution of the survivability. When the uncertainty in position is small as in the bottom case, the distribution for survivability is well centered around 53 %, which is the true value for the survivability at the last waypoint, see Figure 1. When the uncertainty in position is increased, the width of the distribution is increased. This is reflected both in the standard deviation as well as in the entropy, which have both increased.

Figure 3 shows the simulation results of the survivability for the waypoints along the route. At the second waypoints, the aircraft has not passed any threat and the survivability is

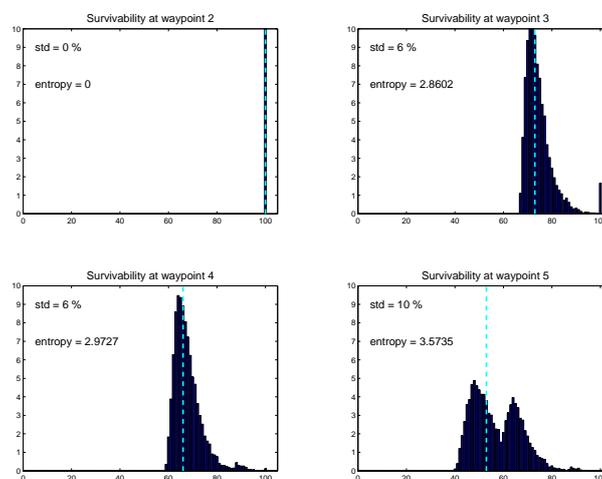


Figure 3: Histogram of the results from Monte Carlo simulations of the survivability for the waypoints along the route. The mean survivability, the standard deviation and the entropy is also displayed. The first and the last waypoints (W1 and W6) have been omitted since these results are equal to the results for waypoint 2 and waypoint 5 respectively.

therefore 100 %. The uncertainty in threat position has not affected the survivability in this case and therefore both the standard deviation and the entropy is zero. By the third waypoint, the aircraft has passed the first threat and the survivability has decreased. Due to the uncertainty in position of the first threat, there is an uncertainty in the survivability value. This is indicated as the spread of the distribution and in the non-zero standard deviation and entropy. At the fourth and fifth waypoint, the uncertainty in survivability has increased since the uncertainty of the positions of the new threats together with the uncertainty of the first threat all contribute to the uncertainty in survivability. This is indicated both in the standard deviation and the entropy for the survivability, which are larger in these cases. It is also interesting to note that the distribution for the fifth waypoint has two peaks.

### 3.3 Comparison of Uncertainty Measures

The simulation results depicted in Figure 2 and 3 show that the uncertainties in position influence the uncertainty in survivability and that this is reflected in both the estimated standard deviation and entropy. In Figure 4 the proposed uncertainty measures are depicted for the different waypoints and different values of uncertainty in position. Both

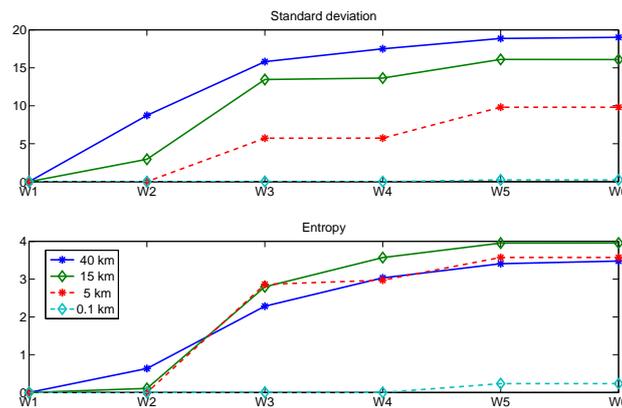


Figure 4: Standard deviation and entropy for all waypoints along the route and different values of the position uncertainty of the threats. Both measures reflect that the uncertainty in survivability is increased when the aircraft flies along the route (higher waypoint number). The highest position uncertainty results in the highest standard deviation. The entropy on the other hand, is larger for position uncertainty of 5 and 15 km than position uncertainty 40 km for all waypoints after the third.

measures are increased when the position uncertainty is increased from 0.1 km to 5 km or 15 km. They also increase when the aircraft flies along the route since the position uncertainty from more threats affect the survivability later in the route. This indicates that both measures can be used for describing the uncertainty in survivability.

However, if the position uncertainty is increased from 15 km to 40 km, the measures are more equivocal. The largest position uncertainty results in the largest standard deviation but not the largest entropy. An understanding of this behavior can be given from Figure 5, which compares the histogram at the last waypoint (W6) for the position uncertainty of 15 km and 40 km respectively. The standard deviation is smaller for the red case, while the entropy is smaller in the blue case. When visually comparing the plots it is not clear which of these cases that are most uncertain. The blue case has a few peaks and especially a large peak at 100 %, which explains why the entropy is smaller in this case. In the red case, the distribution is more gathered around the mean value, which is described by the smaller standard deviation. The standard deviation and the entropy therefore focus on different aspects of the uncertainty and which measure that is most suitable depends on the application.

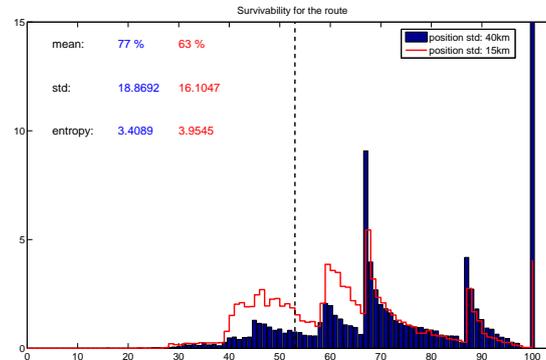


Figure 5: Histogram of the results from Monte Carlo simulations of the survivability for the route. The mean survivability and estimated standard deviation and entropy are also displayed.

## 4 Decreasing Uncertainty

When the uncertainty in survivability is large, it would be desirable to gain more information to decrease the uncertainty. This can be achieved by directing the sensors on the aircraft against the threat that one wishes more information about or send a surveillance patrol to the area before the mission. It is desirable to gather information about the threat that has the largest impact on the uncertainty in the survivability. In this section, we investigate how the uncertainty in survivability will be affected if the uncertainty for one of the threats is reduced.

Figure 6 shows the uncertainty measures for the case when one of the threat's position uncertainties has been reduced. When considering the survivability for the third and fourth waypoint (W3 and W4), it is most desirable to reduce the position uncertainty for the first threat. When considering the whole route, both the standard deviation and the entropy indicate that it is more desirable to reduce the uncertainty of the third threat.

Figure 7 show the distribution of survivability at the last waypoint (W6). It is clear from both the distribution and the uncertainty measures that the reduction of position uncertainty for threat 2 only has a small influence on the uncertainty in survivability. This is not surprising since threat 2 only has a small impact on the survivability of the total route. On the other hand, the reduction of position uncertainties for both threat 1 and threat 3 largely influence the uncertainty in survivability, resulting in other shapes of the distributions in Figure 7. The reduction of position uncertainty for threat 3 even changes the distribution from having two peaks to only one peak. From the distributions it seems to be most desirable to reduce the position uncertainty for threat 3, which is also indicated in both the entropy and standard deviation.

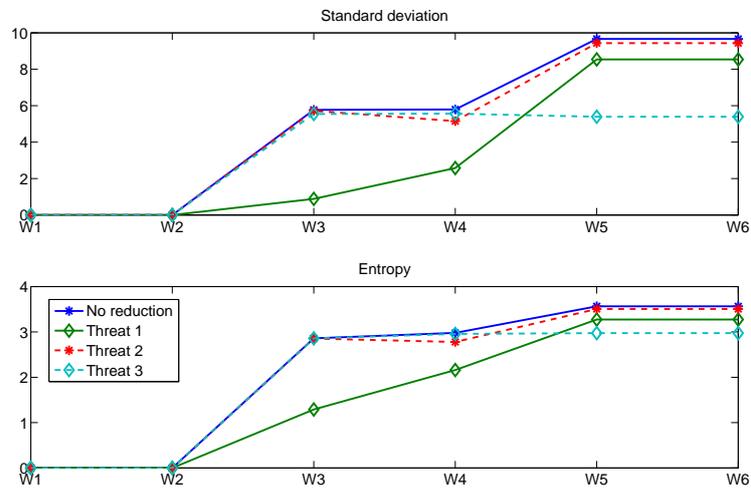


Figure 6: Standard deviation and entropy for the waypoints along the route where the uncertainty in position for one of the threats has been reduced from 5 km to 1 km (std). The case when all threats have the higher uncertainty is also displayed for comparison.

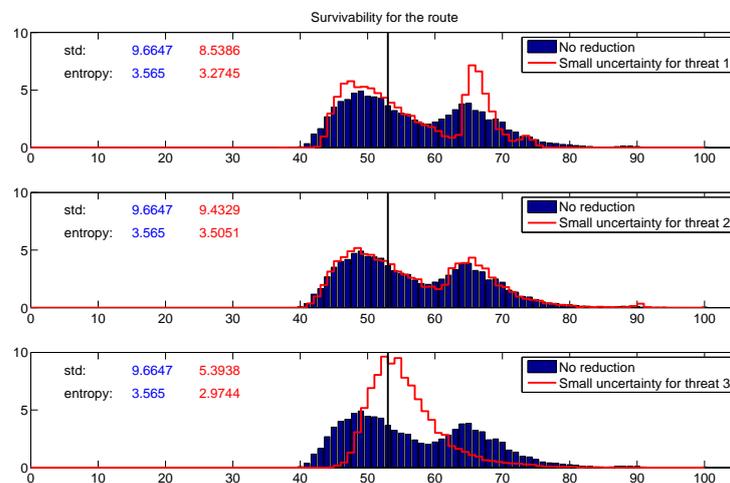


Figure 7: Histogram of the results from Monte Carlo simulations of the survivability for the route, where the position uncertainty for one of the threats has been reduced respectively. The case with no reduction is also displayed for comparison.

## 5 Discussion and Issues for Future Work

The aim of sensor management is to optimize the use of the sensor resources in order to gather the most valuable information, in this case the information that minimizes the uncertainty in survivability. This can be achieved by comparing the present distribution for the survivability with the anticipated distributions that can be achieved if different sensors tasks take place. The sensor task that result in the least uncertain distribution should then be applied. The investigations in Section 3 show that both the standard deviation and the entropy are possible to use for describing the uncertainty of a distribution. However, the measures capture different characteristics of the uncertainty in the distributions and in some situations the distribution with the smallest standard deviation might not be the same as the one with the smallest entropy, see for instance Figure 5. On the other hand, it is not clear which distribution that is least uncertain in this case. It is up to the designer of a sensor management system to determine which of these measures that is most suitable.

There are other issues that also need to be addressed when designing a sensor management system for a fighter aircraft that may influence the choice of uncertainty measure. The following of this section discusses a few of them.

### **Undetected Threats.**

A sensor management system can not only focus on keeping track of the detected threats in the area, but must also search for undetected threats. If many missions have recently been performed in the area, there might be a good appreciation of the number of threats in the area. On the other hand, if only a few missions have been conducted, there are probably more threats than the detected ones.

### **Multi-Objective Sensor Management.**

Section 1 describes that the fighter pilot has three objectives when flying the aircraft, namely flight safety, mission accomplishment and combat survival. These objectives need to be reflected in a sensor management system. The system can not only focus on minimizing the uncertainty in survivability, but must also provide the pilot with information for flight safety and mission accomplishment. Information requests regarding other objectives must therefore be handled and prioritized against the objective of minimizing uncertainty in survivability.

### **Real-Time Calculations of the Uncertainty Measure.**

Sensor management in a real-time application requires that the calculations of the relative gains of different sensor tasks are possible to calculate in real-time. When using an uncertainty measure for describing this, it would be desirable to have an algorithm that can calculate the uncertainty measure or an approximation of it, without Monte Carlo simulations. Since the real value of the measure is of less importance than the relative difference between two values, it might be possible to use rough approximations. If it is not possible to derive an analytic expression, it should be investigated if the number of Monte Carlo simulations can be reduced but still produce a useful approximate of the uncertainty measure.

### **Information Need versus Emission Control.**

An active sensor, such as radar, emits energy which might be detected by threats. As pointed out in [NN00], there can therefore be requests to restrict the sensor usage to achieve low probability of intercept and to increase survivability. In particular, if the threats are equipped with radar warning system, it is desirable to minimize the energy emission from the aircraft. However, this conflicts the aim of gaining more information. It is therefore important to find a balance between gathering information and remain undetected. The weight of these two objectives may change over the mission.

### **Team of Fighter Aircraft.**

Fighter pilots usually operate within teams of several aircraft. This enables them to share data and information among the aircraft [HENF10]. Extending the sensor management problem discussed in this paper to a team of aircraft offers the opportunity to distribute the sensor measurement among the aircraft in the team. If passive sensors are used, the fusion of sensor measurements from different aircraft can improve the position estimate of the threats.

## **6 Conclusions**

A fighter pilot flying a mission has access to a lot of information from data bases and sensors. This information is used by the pilot to analyze the situation and achieving situation awareness. A sensor management system can control the sensors in order to gather the information that is most valuable for the pilot's understanding of the situation. An important component is the estimate of the aircraft's survivability, i.e. the probability that the pilot can accomplish his mission without being hit by enemy fire. One of the objectives for the sensor management system should therefore be to reduce the uncertainty in the survivability estimate by gathering more information about the threats in the area. In this paper we utilize the survivability model presented in [ENNW11] and perform Monte Carlo simulations with the model in order to investigate two possible measures of uncertainty; standard deviation and entropy. The simulation results show that both measures reflect the uncertainty, even though there are cases were they do not coincide. It is therefore likely that these measures of uncertainty can be used for sensor management.

Furthermore, we discuss a few issues that need to be addressed in the design of a sensor management system in a fighter aircraft, namely:

- Detect unknown threats contra gathering more information about known threats
- Multi-objective information requests
- Real-time calculations of the uncertainty measure
- Information need contra emission control
- Cooperating information sharing within a team of fighter aircraft

## Acknowledgement

This research has been supported by The Swedish Governmental Agency for Innovation Systems (Vinnova) through the National Aviation Engineering Research Program (NFFP5-2009-01315), Saab AB and the University of Skövde. We would like to thank Per-Johan Nordlund (Saab AB, Aeronautics, Linköping), Tove Helldin and Ronnie Johansson (University of Skövde, Skövde) for their suggestions and fruitful discussions.

## References

- [BDDR<sup>+</sup>09] M. Ben-Daya, S.O. Duffuaa, A. Raouf, J. Knezevic, and D. Ait-Kadi. *Handbook of Maintenance Management and Engineering*, chapter 3. Springer Verlag, 2009.
- [Blo84] G. Blom. *Sannolikhetsteori med tillämpningar*. Studentlitteratur, Lund, Sweden, second edition, 1984.
- [BRM88] S.G. Bier, P.L. Rothman, and R.A. Manske. Intelligent Sensor Management for Beyond Visual Range Air-to-Air Combat. In *Proceedings of the IEEE National Aerospace and Electronics Conference, NAECON*, pages 264–269. IEEE, 1988.
- [ENNW11] T. Erlandsson, L. Niklasson, P-J. Nordlund, and H. Warston. Modeling Fighter Aircraft Mission Survivability. In *14th International Conference on Information Fusion (FUSION 2011)*, 2011. Chicago, United States.
- [HENF10] T. Helldin, T. Erlandsson, L. Niklasson, and G. Falkman. Situational adapting system supporting team situation awareness. In *Proceedings of SPIE, the International Society for Optical Engineering*, volume 7833. Society of Photo-Optical Instrumentation Engineers, 2010.
- [Mal95] R. Malhotra. Temporal Considerations in Sensor Management. In *Proceedings of the IEEE National Aerospace and Electronics Conference, NAECON*, volume 1, pages 86–93. IEEE, 1995.
- [NN00] G.W. Ng and K.H. Ng. Sensor management-what, why and how. *Information Fusion*, 1(2):67–75, 2000.
- [OHJ<sup>+</sup>04] W.L. Oberkampf, J.C. Helton, C.A. Joslyn, S.F. Wojtkiewicz, and S. Ferson. Challenge problems: uncertainty in system response given uncertain parameters. *Reliability Engineering & System Safety*, 85:11–19, 2004.
- [Sch01] A. Schulte. Mission Management and Crew Assistance for Military Aircraft - Cognitive Concepts and Prototype Evaluation. Technical report, ESG - Elektroniksystem - und Logistik -GmbH Advanced Avionics Systems, 2001. Paper presented at the RTO Lecture Series on "Tactical Decision Aids and Situational Awareness" and published in RTO-EN-019.
- [XS02] N. Xiong and P. Svensson. Multi-sensor management for information fusion: issues and approaches. *Information fusion*, 3(2):163–186, 2002.
- [YG05] R.D. Yates and D.J. Goodman. *Probability and Stochastic Processes: A Friendly Introduction for Electrical and Computer Engineers*. John Wiley & Sons, 2005.