Some Applications of Graph Transformations in Modeling of Mechanical Systems

Stanisław Zawiślak, Łukasz Szypuła, Mirosław Myśliwiec and Adam Jagosz

Department of Fundamentals of Machine Building
University of Bielsko-Biała, Poland

Abstract: In the paper, some graphs representing mechanical systems are analyzed. The transformations of graphs representing a gear and a truss are considered. These transformations of the graphs allow for derivation of simplified calculation courses in some cases e.g. gear ratios or forces. Furthermore, these simplifications are mutually connected with adequate changes of the graph or sub-graphs in step-wise manner taking into account the functional schemes of the artifacts. The derived calculation methods utilize other algebraic objects associated with a graph e.g. cut matrices and sets of fundamental cycles.

Keywords: gear functional scheme, Hsu's graph, f-cycle, cut matrix, kinematical analysis

1 Introduction

Graphs are used as models of versatile technical systems e.g. electrical and electronic systems, railways and road networks, phone networks and mechanical systems. The last mentioned area of application is relatively new and it is relatively narrowly known. Nevertheless the achievements within this area of investigation are crucial for AI applications in mechanical design or AI-aided design [9]. The most essential and simultaneously wide introduction to the graph representation of mechanical systems can be found in books Tsai [11], Rudolph [6] and Kaveh [4] describing some different aspects, respectively. The valuable contributions to the discussed field have been done by Hsu [3], Shai [9], the author [15-19] and many others.

Graph transformations are used in engineering applications mainly in civil engineering [1,2,10] but recently this tool was also used in mechanical engineering [5,7,8,13]. The graphs' application in civil engineering focus mainly on a layout of civil engineering structures e.g. trusses, buildings or floor arrangements in buildings. In paper [5], the reference review connected with an application of graph-grammars in mechanical design has been made. The described methodology was used for generation of new designs of gears i.e. their functional schemes. In paper [7], the synthesis of mechanisms based upon an application of graph grammars is presented. The task of synthesis and enumeration of all possible designs of a particular mechanical artifact can also be performed by means of the graph-based approach [11]. Firstly, graphs are used e.g. for encoding of a functional scheme of a planetary gear or a geometrical structure of a truss. Secondly, base on some algebraic objects related to the graphs (e.g. matrices, polynomials or matroids [15]) the calculation methods are derived and used. The transformations of the graphs assigned to planetary gears are shown in the paper. Some adequate transformations are connected with the changes of drives and the related changes of passage of a rotational movement and power throughout a gear. Simultaneously these transformations cause simplification of adequate equation systems in an automatic way. The whole process makes possible to analyze simplified functional schemes and it allows for derivation of relevant simplified kinematic equation systems for consecutive considered work modes in case of the automatic gear boxes. The goal of this paper is to show which graph transformation are used in modeling of gears and trusses as well as what is the mechanical interpretation of these transformations which – moreover - are different from other approaches shown in the cited references. It is worth to underline that in recent years, it was very rare to analyze versatile mechanical artifacts in the light of common graph transformations approach. On the contrary, usually just single objects were considered or other aspects of graph models were highlighted. Moreover, it has to be added that O. Shai [9] compared several graph-models of artifacts from the AI knowledge transformation perspective.

2 Graphs as models of mechanical systems

The possibility of representation of a mechanical system *M* by means of a graph *G* consists in simplification and representation of the system *M* by means of relations between its elements. These relations can be then turned into graphs where elements of relations are presented as edges with adequate weights. There are versatile graph representations of a planetary gear [15]. The review of some more frequently used approaches is given in [18]. Other graph-based methods of modeling of mechanical systems are described in works [9, 11]. A graph $G(V, E W)$ is a weighted graph, where: $|V| = n$, $|E| = m$ and a function $W: E \rightarrow \{set\ of\ weights\}$. The function W as well as the set of weights depend on a considered artifact and a considered problem. A single weight is usually assigned to a single edge e.g. see explanation given to Fig. 1. However in some cases - when advanced mechanical analysis is performed upon a graph model of an artifact (i.e. a truss) - several weights are assigned to a single edge. For example, in case of a truss (Fig. 2) when stresses are analyzed every edge represents a rod made of metal - therefore some weights are as follows: cross-section area of the bar [m^2], force acting along the bar [N], physical and mechanical properties of the bar material e.g. density [kg/m^3] etc. [19]. Moreover, sometimes instead of weights the different line types are used for edges pictograms aiming for easiness in interpretation of a weighted graph. In Fig.1 the functional scheme of a gear and the assigned weighted graph are presented. Two parallel short lines perpendicular to the main axis indicate that scheme is symmetrical but the symmetrical image under the axis is omitted. Planets *2* and *7* in Fig. 1a can look like geared wheels presented in Fig. 1c, arm *1* is omitted. Meshing of elements *6* (planet) and *3* (wheel with internal toothing) can look like it is shown in Fig. 1d. A relation between a pair of two gear elements i.e. the fact that two geared wheels are in mesh - is shown in a graph as stripped-line edge. For example: geared wheels *2* and *7* (so called planets) in Fig 1a are turned into the vertices *2* and *7* in the graph in Fig. 1b. Because these planets cooperate in a mechanical sense (their teeth are especially in contact; generally speaking in mesh) therefore the edge $\{2,7\}$ is drawn as a stripped one. In case of a pair of gear elements: arm and planet – their adequate vertices are connected via continuous line starting from polygon e.g. element *1* and *2* (arm and planet) are represented as vertices *1* and *2* connected via the edge $\{1,2\}$ drawn as continuous. The rotational axis of the planet is fixed to the arm. The arm assures a constant distance between the main gear axis and the axis of the planet allowing for their mutual rotational movement. We can consider different elements as input and output of the gear e.g.: 5-1, 1-3 or 5-3, respectively. The Hsu's method of assignment of a graph to a gear scheme has been utilized. The method can be summarized as follows: vertices of graphs and adequate gear elements are notated by means of the same labels i.e. natural numbers. The relations between elements of the gear are considered as follows:

- all pairs which rotate around the same main axis (rotational pairs) are represented by vertices of a (shaded) polygon e.g. *1, 3, 4* and *5*. Therefore we have pairs: *{1,3},{1,4}, {1,5}, {3,4}, {3,5}* and *{4,5}*. The whole full graph being a subgraph of the considered graph of the planetary gear represents the whole set of these rotational pairs (6 in case of rectangle, 10 in case of pentagon etc.). The rule was introduced that this full graph is replaced in figures by a shaded polygon. This approach allow for achieving a more readable form of the graph. This approach is dedicated only to the illustrative form and in further detailed considerations connected e.g. with distinguishing of cycles (called here f-cycles [11]) all edges of a full graph are theoretically considered despite that they are hidden in the drawn polygon;

a) b) c) d)

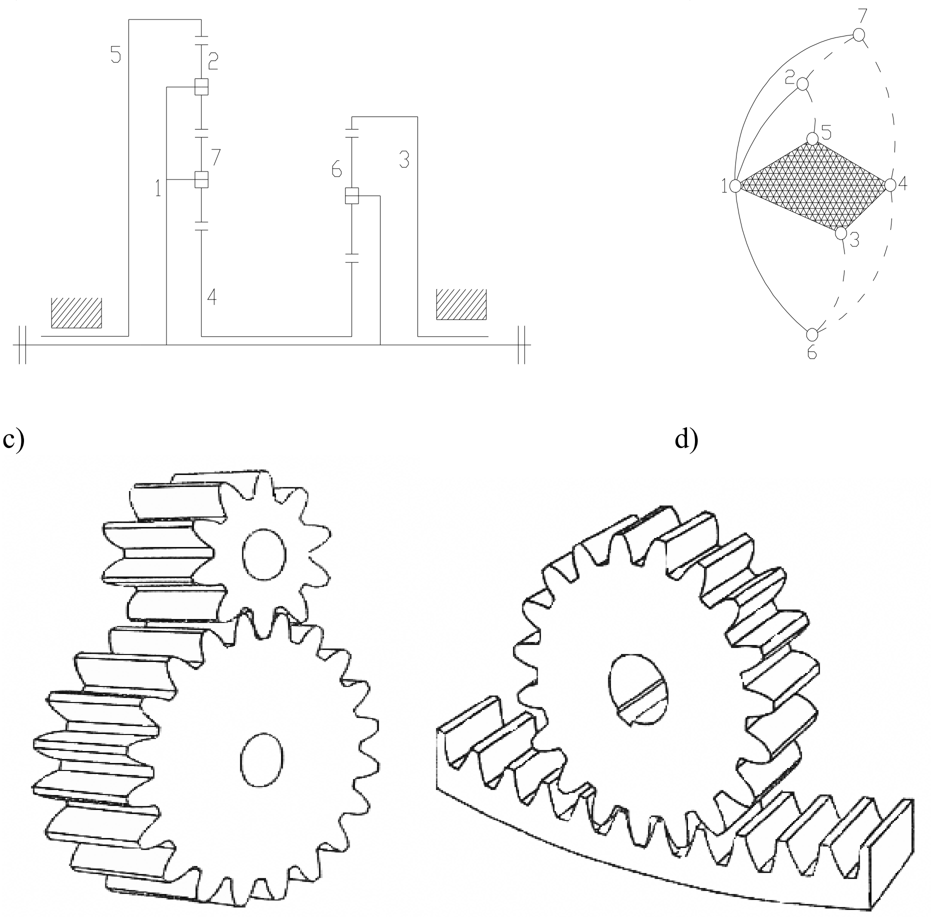


Fig. 1. Planetary gear (a), its Hsu's graph (b), external (c) and internal (d) meshings

- pairs of elements being in mesh (co-operation of two toothed wheels, geared wheels) are represented via stripped lines (edges) e.g. *{4,6}, {3,6}, {4,7}, {5,2}* and *{2,7}*. The meshing between the planets *2* and *7* is external (Fig.1c) whereas the meshing between wheels *2* and *5* is internal (Fig.1d) what will have respectable meaning in writing the equations describing the kinematics of a gear wheel but it is not encoded in a graph explicitly. Internal meshing means that the wheel *5* has its tooth inside a toothing ring of the element *5*;

- rotational pairs "planet wheel – arm (carrier)" are represented via continuous edges i.e.: *{1,7}, {1,2}* and *{1,6}*.

The powerfulness of the linear graph representation of mechanical systems consist in usage of versatile algebraic structures connected with the graph and derivation of some calculation methods utilizing these algebraic objects (chapters 3.2 and 3.3, below). On the contrary, bond graphs based methods are assigned to a mechanical system [15] in one way only. It restricts their applicability. Tasks which can be done using linear graph-based models are as follows:

- analysis of gear ratios [11],
- solution of a reverse problem i.e. assignment of a functional scheme to a gear graph [5, 17],
- enumeration of all gear schemes fulfilling a particular constraint i.e. having less then e.g. *7* or *8* elements [11] – till know it has been possible only based graph-theoretical approach,
- calculation of forces in a truss loaded by external discrete forces (acting in nodes) [14],
- checking a stiffness of a truss [4] and many others [11].

In the underneath consideration e.g. a cut matrix for trusses and f-cycles for gears graph models will be applied in adequate calculations. A truss is a structure made of rods. Neglecting the type and dimensions of joints the simplified model is obtained. Such a model is frequently used in an introductory engineering calculations. At the beginning an assignment of a graph to a truss is done in a natural way: nodes of truss are converted into vertices and bars into edges, respectively. The model and the transformation steps which simplify some calculations are described underneath. Transformations of graphs which are assigned to mechanical systems have several goals:

- derivation of calculation methods in simple manner taking into engineering knowledge transformed into a field of graphs (obtained methods are equivalent to the traditional ones),
- automation of calculation courses and
- constructing a design form in an algorithmic way via step by step performed routine.

3 Transformations of graphs being models of mechanical systems

Graph representations of automated gear boxes can be used for an automation of a ratio calculation as well as some other tasks. The essence of transformation relays in every case in different task:

- creation of a functional scheme step by step – several mutually related facts about graph are taken into account,
- creation of a system of equations (describing e.g. a truss) in algorithmic way – assuring the proper arrangement of these equations,
- simplification of a gear functional scheme and simultaneously simplification of an adequate graph. It also helps in an analysis of the direct path of passing a rotational movement from the input to the output of a gear. Moreover it allows for a ratio calculation for every drive upon a simplified subgraph. Underneath, firstly, the problem of creation of a functional scheme of an exemplary gear is analyzed.

3.1. Building of a gear functional scheme based upon its graph

The problem of conversion of a graph being a model of a mechanical system into its functional scheme has not been fully solved until now [7,11] especially in a fully automatic or algorithmic way. Underneath, the proposal is formulated how to interpret the consecutive phases of creation of a functional scheme of a gear as a process of expanding a subgraph which evolves from an initial chosen vertex up to the whole graph. The task of drawing (in the graph) of particular edges and vertices (and drawing parallely adequate mechanical elements) is performed in algorithmic way analyzing a graph and a scheme in a cross reference manner – one choice implies the next one in accordance with clear, unequivocal rules.

Assignment of the graph presented in Fig. 1b to the scheme shown in Fig. 1a is illustrated in Table 1 (in what follows denoted by T1). The procedure of assignment of a functional scheme to a graph can be considered in following steps:

(i) choice of a vertex in a polygon – the vertex *1* was chosen. It implies that all the edges starting from *1* are considered simultaneously i.e. *{1,2}, {1,6}, {1,7}*. They are drawn as bold lines in box T.1(1,2). The interpretation of the scheme in box T.1(1,1) is as follows: *1* – is the main axis of a gear. Moreover, *1* – is a carrier for the planets *2,6* and *7* because the graph edges are drawn by means of continuous lines,

(ii) edge *{2,7}* is a stripped line therefore planets *2* and *7* are a pair of geared wheels in mesh /see: first row of the table, box T1(1,2)/,

(iii) choice of the vertex *3*. It is the second consecutive vertex of the polygon therefore its representative i.e. gear element *3* (geared wheel with an internal toothing) rotates around the main axis. The edge *{3,6}* is drawn as a stripped line – see box T.1(2,2) – therefore elements *3* and *6* are in mesh, element *3* is added in the created functional scheme - box T.1(2,1),

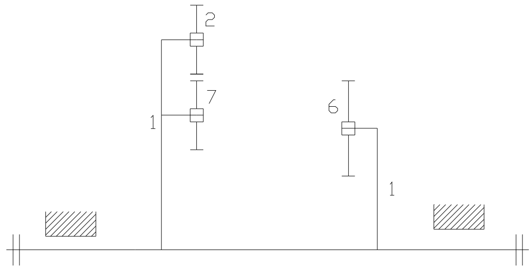
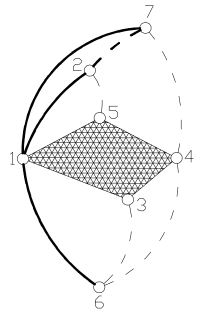
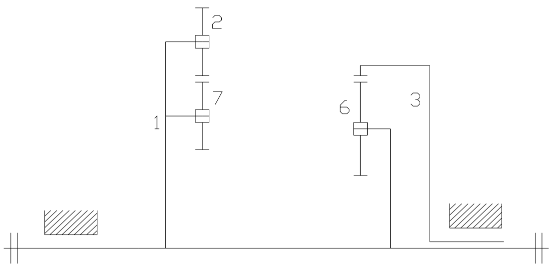
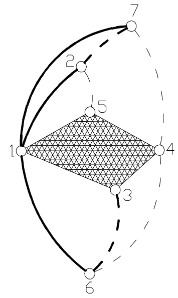
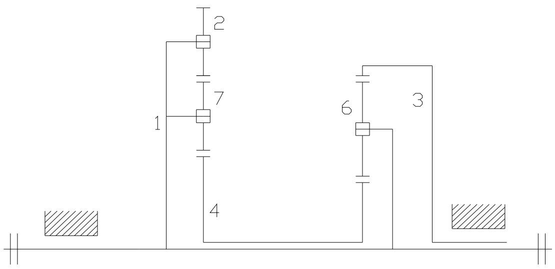
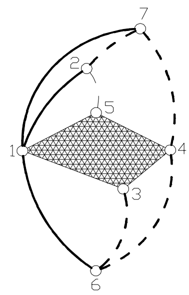
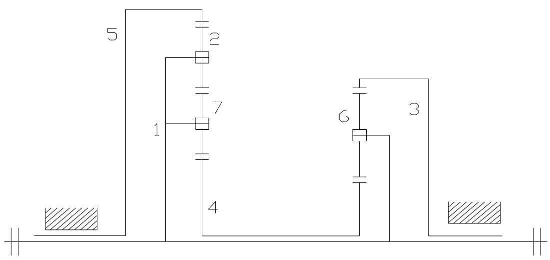
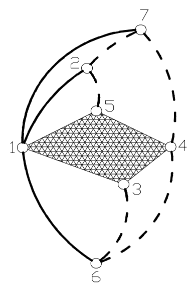
(iv) choice of the vertex *4*. It is the polygon node so the element *4* rotates around the main axis, furthermore the edges *{4,6}* and *{4,7}* are drawn as stripped lines - box T.1(3.2). Therefore the element *4* is in mesh with two planets simultaneously – in mechanical sense it means that it is so called sun. This fact is illustrated in box T.1(3.1),

(v) choice of the vertex *5*. It is the last vertex of the polygon - box T.1(4.2). So, element *5* rotates also around the main axis. The edge *{5,2}* indicates that the elements *2* and *5* are in mesh - box T.1(4.1). The functional scheme has been fully done.

The transformation of the graph is here shown symbolically as turning more and more parts of the graph into bold lines. So transformation of a subgraph is really simple: adding graph elements one after another but simultaneously the functional scheme has to be built in accordance with the graph and the mechanical point of view. Finally the bold subgraph is turning into a final complete graph fully drawn as bold - so the procedure is finished. This procedure is needed for a task of creating of a family of design solutions of gears. The properties of graphs representing gears have been formulated [11]. Upon generating of the family of the graphs which fulfil these conditions [11] – the atlases of designs are built i.e. the sets of functional schemes of particular gears. However in many papers the process is finished on the phase of graphs generation believing that the final step could be done by a reader.

Therefore performance a conversion: “graph-functional scheme” described above – causes that the family of designs is understandable for engineers. This phase was frequently omitted in papers dealing with graph models of gears but sometimes it had caused that the existence of some mistakes [18] was not revealed. Only analysis of the gear scheme by an experienced engineer allows for full analysis of the machines from mechanical point of view. So, it seems that a fully algorithmic procedure has to be prepared applying an expert system. It exceeds the range and aim of the present paper.

Table 1. Building of the functional scheme upon the gear graph

Graph Representation of planetary gear		
	Functional scheme	Graph representation
	1	2
1		
2		
3		
4		

Here only the rough idea of transformation of the gear graph is given. Till now, it has not been turned into the computer program. More automatic and computerized version is described in [5].

3.2. Truss analysis

Second analyzed exemplary mechanical system is a truss – considered in a simplified version formulated in a previous chapter for statically determined trusses. We focus our attention on a problem of forces calculation but many other tasks can be also performed [4, 9]. The applied procedure of assignment of a graph to a truss is as follows:

(a1) truss is a linear graph itself, let's number/label its vertices by natural numbers from *1* to *n* or n capital letters *A, B, C*, etc.,
(a2) turning a graph into a directed graph is performed via introducing arcs according to the rule that direction of an arc is established from a vertex of a lower number to a vertex of greater number or taking into account a sequence of characters of the Latin alphabet,
(a3) additional tree is introduced which allows for systematic assignment of the cut matrix.

The discussed calculation method does not depend on labeling (numeration) of the graph vertices and the tree assignment where the root of the tree can be placed outside the existed graph or one of its vertices. The assignment of direction can be done in arbitrary way (it arises directly from numeration). Every arc symbolizes the direction of acting force. If the resulting forces are obtained as negative then it means that the direction of a force is reverse. If all forces are positive it means that all directions of acting forces are the same as direction of graph edges (arcs). The exemplary truss and its graph is presented in Fig. 2 a-f.
The derived method of the forces calculation [14] can be described in the following steps:

(s1) 2D truss is a graph itself considering nodes as vertices and bars(rods) as edges,
(s2) choice the special reference vertex, frequently the left hand, bottom node of the truss is chosen, enter the vertices numbering in an arbitrary way, it represents an external loading,
(s3) entering of the orientation of edges: from vertices of lower to upper number label, the method does not depend on numbering of vertices,
(s4) creation of the special tree - which branches connect the chosen vertex with all other ones creating the original truss, it is an additional (theoretical) tree allowing for further steps. The orientation of branches is from the chosen vertex to the truss nodes. Additional remark: the introduced tree allows for performance of all other tasks in an algorithmic manner as well as it has mechanical meaning – allows for inserting external forces to the calculation course,
(s5) creation of the cut matrix (${}_2B$) of this new graph (Fig. 2f) – placing the columns associated with the branches of the tree at the beginning. It is a well-known fact from graph theory: elements of the cut matrix are as follows: *{0, 1, -1}* what is relevant to the fact that a considered edge does not belong, it goes inside or it goes outside of a particular cut, respectively. Moreover – the cuts are generated by the tree branches, therefore the submatrix corresponding to them is a E matrix. The cut matrix for the considered truss (Fig. 2a) is presented by the formula (1):

$$
{}_2B = \begin{array}{c} \\ I \\ II \\ III \\ IV \\ V \\ VI \end{array}
\begin{array}{c}
\begin{array}{cccccc|ccccccccc} I & II & III & IV & V & VI & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{array} \\
\left[\begin{array}{cccccc|ccccccccc}
1 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & -1 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & -1 & -1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1
\end{array}\right]
\end{array} \tag{1}
$$

where: *I, II, ..., VI* in the upper row denoted additional tree arcs and in the initial column – cuts connected with the end points of these arcs, respectively,

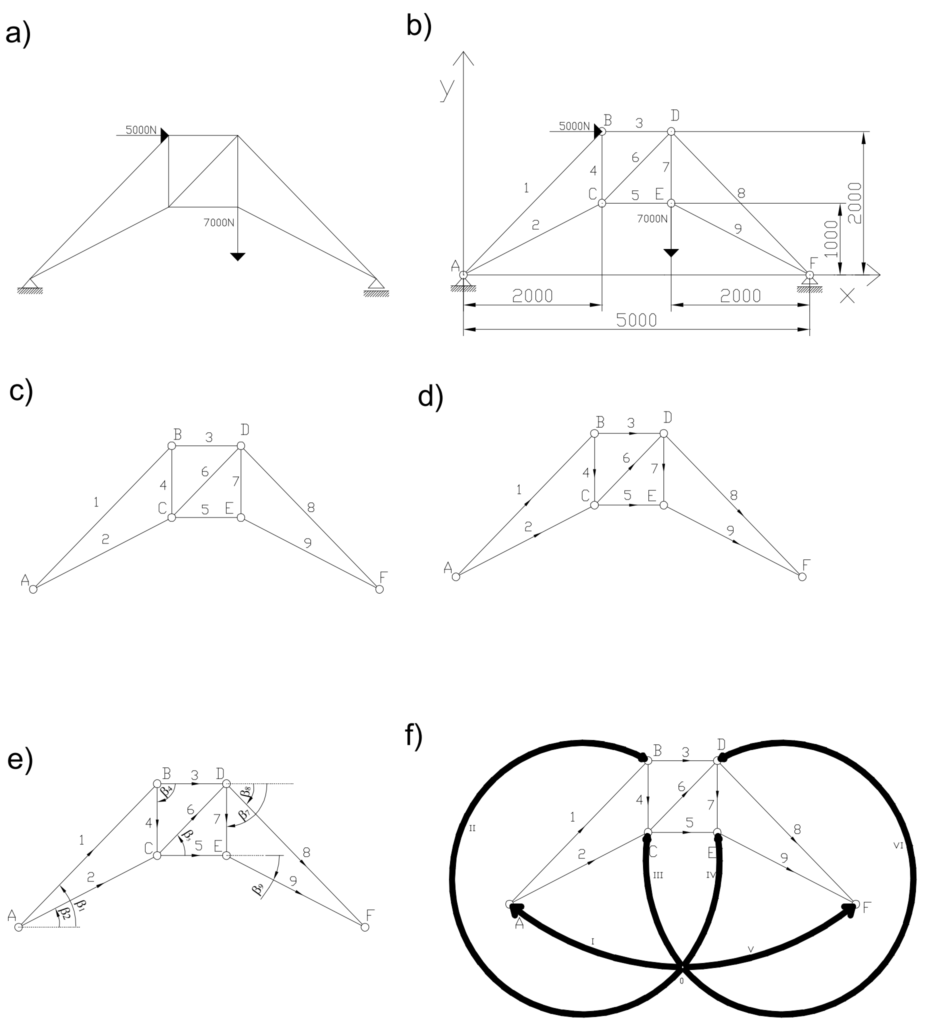


Fig.2 Truss and its graphs, (a) truss, (b) dimensions, (c) linear graph, (d) directed graph, (e) angles of truss rods ($\beta_3 = \beta_5 = 0$), (f) graph with an additional tree

(s6) creation of so called generalized (or rearranged) cut matrix where elements *0,1* and *–1* are replaced by matrices of rank *2 x 2* where these elements are doubled on the main diagonal and the remaining elements are zero (it is equivalent to consider x and y components separately). After performance of these actions, the first submatrix corresponding to branches of the tree is still a unit matrix. Therefore, the cut-matrix can be divided into two sub-matrices, first of them is the matrix representing the branches of the tree i.e. E matrix (with 1 on the main diagonal). Finally, the remaining submatrix (i.e. ${}_{20}\boldsymbol{B}^{R}$) is used for creation of the system of equations,
(s7) creation of trigonometry functions (or transformation) matrix – collecting the angles of inclinations of forces in the nodes and the rods according to the orthogonal coordinate system connected with some set of vertices, the original of the introduced co-ordinate system is placed in the point chosen in step (s2). It allows for considerations of X and Y components of forces. The matrix is denoted by $\boldsymbol{C}_{\beta}$,
(s8) creation of the vector matrix of external forces $\boldsymbol{F}_{Z}$**'**,
(s9) creation of the system of equations describing a static analysis of the statically-determined truss, additionally the rows connected with fixed supports has to be crossed out. In mechanical sense it means that reaction of the ground are not calculated via this approach,
(s10) solution of the system by finding the inverse matrix for the matrix generated in step (s7),
(s11) finding the wanted forces in the rods – collected in matrix $\boldsymbol{S}$ – using the formula (2):

$$S = [{}_{20}\boldsymbol{B}^{R}\ \boldsymbol{C}_{\beta}]^{-1}\ \boldsymbol{F}_{Z}\text{'} \tag{2}$$

where:
S – forces in bars,
${}_{20}\boldsymbol{B}^{R}$ – special matrix connected with the transformed graph representation of a truss,
$\boldsymbol{C}_{\beta}$ – matrix of cosinus and sinus functions, upon the geometrical dimensions of the truss elements,
$\boldsymbol{F}_{Z}$**'** – external forces.

Preciseness of the method depends on the procedure of creating a reverse matrix. In many cases the reaction of the ground are needed for mechanical analyses, so they can be additionally calculated upon classical attitude. It is a slight drawback of the graph-based approach.
External forces are described by formula (7), underneath.
The analysis of the truss is performed according to the above presented steps (s1)-(s11). Step (s6) consists in inserting the formulas:

$$1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix};\quad -1 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix};\quad 0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{3}$$

The matrix ${}_{20}B$ has the following form:

$$
{}_{20}B = \begin{bmatrix}
-1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1
\end{bmatrix} \tag{4}
$$

Matrix of trigonometric functions is as follows:

$$
C_\beta = \begin{bmatrix}
\frac{\sqrt{2}}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{\sqrt{2}}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{2\sqrt{5}}{5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{\sqrt{5}}{5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{2}}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{2\sqrt{5}}{5} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{5}}{5}
\end{bmatrix} \tag{5}
$$

$$
F'_z = \begin{bmatrix} 5000 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -7000 \\ 0 \end{bmatrix} \tag{6}
$$

For example: the rod *1* is inclined by 45^o. Therefore sin 45^o = cos 45^o = *sqrt(2)/2*; these are elements of the matrix C_β i.e.: C_β *(1,1) and* C_β *(2,1)* . The results obtained upon the formula 2 are as follows - forces in [N] in rods are: *S = [9337, -12989, 11 600, -6600, - 12400, 1131, -*

13200, 17536, - 13864], respectively. In the considered method the data representing the truss structure and the dimensions are separated in two distinct matrices. It is very important feature which is not characteristic for some other traditional methods. This fact allows also for e.g. evolutionary approach to truss design and optimization [19] what was done in the master thesis of the co-author (A. Jagosz). It was also shown that the graph method gives the same results as e.g. traditional one as well as achieved by means of FEM. Underneath (a) is used instead of Fig 2a. The transformations in Fig. 2 are as follows: (a)&(b)→(c) modeling in natural way – considering a structure as a graph; (c) →(d) according to the rule "a1", (d) →(e) calculation the angles [with a direction] in algorithmic way based upon vector and scalar products of adequate vectors; (d) →(e) adding tree according to the rule "s4". The final graph is suitable for performing of the force calculations.

3.3. Analysis of gear ratios

Third considered problem is calculation of ratios of an automatic gear box. The scheme of the gear can be found in document [12]. The gear consists of 6 elements: geared wheels, planets and carriers, furthermore it is equipped in the system of two brakes (Br-1,Br-2) and two clutches (Cl-1, Cl-2). Activation of some sequences of these control elements allows for changing the drives. In Table 2, the sequences of the control elements and the graphs describing the obtained mechanical systems are listed. Here, Hsu's graph is utilized but the authors of [12] Freudenstein's graphs had used. Due to the fact that here we focus our attention on graph transformations only the graph-theoretical aspects are discussed. The applied method of graph modeling – i.e. Hsu's graph as well as the applied notation are different than in [12], however the same results were obtained (after some recalculations). In [12] some different rules for assignment of negative elementary ratios had been utilized. The consecutive sequences of the control elements i.e. clutches and brakes causes allow that different drives are obtained (different output rotational speeds). The exemplary transformed graphs assigned to the available drives are shown in Table 2. The graph-based method of gear ratio calculation consists of the following steps: (st1) assignment of a graph to a gear; (st2) distinguishing of so called f-cycles – where every f-cycle contains a stripped line edge [i.e. two elements of gear are in mesh, their codes are assigned to the stripped line-ends]; (st3) derivation of the system of equation describing the kinematics of gear elements; (st4) solution of the system finding a searched ratio. In step "(st3)" every equation in the system is created according the algorithmic rule for dealing with indicators.

Therefore, for every f-cycle $(i,j)k$ the equation of f-cycle can be written automatically in the following form:

$$\omega_i - \omega_k = \pm N_{j,k} (\omega_j - \omega_k) \tag{7}$$

where:

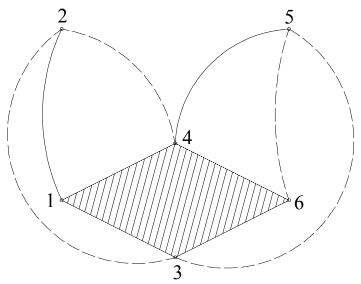
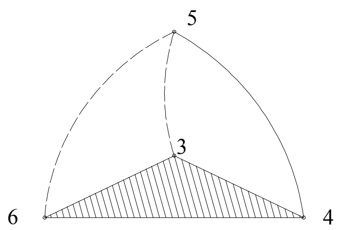
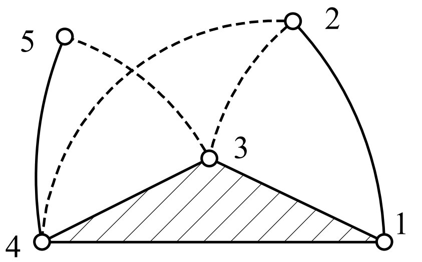
ω_i – rotational speed of the element i,

$N_{j,i}$ – ratio; $N_{j,i} = D_j / D_i = z_j / z_i$, $z_j>0$, $z_i >0$ in all cases; D and z with adequate indicators describes diameters and numbers of tooth on the particular geared wheels i and j, respectively,

Sign + for internal gearing, sign – otherwise.

Elements i and j are in mesh, element k is a carrier (an arm). The graph transformations are here connected with changes of the adequate functional schemes where the so called redundant elements are omitted. Moreover transformed graphs have different sets of f-cycles which fully describe the kinematics for the consecutive stages (writing equations based on the formula (7)). The adequate sequences of the control elements (clutches, brakes) are listed in column 1 in Table 2.

Table 2. Transformed Hsu's graphs representing the available drives of the automatic transmission [12] and allowable adequate kinamatic analyses

	Activated control elements	Hsu's graph representing the adequate gear version - performing its movements via other rotating elements	Derived equation systems – describing kinematics of the gear mode
	1	2	3
1	Cl-1, Br-1 1-st drive	 F-cycles: (2,4)1; (3,5)4; (2,3)1 and (5,6)4.	$\begin{cases}\omega_2-\omega_1=+N_{42}(\omega_4-\omega_1)\\ \omega_3-\omega_4=-N_{53}(\omega_5-\omega_4)\\ \omega_2-\omega_1=-N_{32}(\omega_3-\omega_1)\\ \omega_5-\omega_4=+N_{65}(\omega_6-\omega_4)\end{cases}$ $\omega_1=0$ Solution: $\frac{\omega_4}{\omega_6}=\frac{N_{32}N_{65}}{N_{32}N_{65}+(N_{42}+N_{32})N_{35}}$
2	Cl-1, Br-2 2-nd drive	 F-cycles: (3,5)4 and (5,6)4.	$\begin{cases}\omega_3-\omega_4=-N_{53}(\omega_5-\omega_4)\\ \omega_5-\omega_4=+N_{65}(\omega_6-\omega_4)\end{cases}$ $\omega_3=0$ Solution: $\frac{\omega_4}{\omega_6}=\frac{N_{65}}{N_{65}+N_{53}}$
3	Cl-1, Cl-2 3-rd drive	Direct passage of rotational movement	$\omega_4/\omega_6=1$
4	Cl-2, Br-1 reverse drive	 F-cycles: (2,4)1, (2,3)1 and (3,5)4.	$\begin{cases}\omega_2-\omega_1=+N_{42}(\omega_4-\omega_1)\\ \omega_2-\omega_1=-N_{32}(\omega_3-\omega_1)\\ \omega_3-\omega_4=-N_{53}(\omega_5-\omega_4)\end{cases}$ $\omega_1=0$ Solution: $\frac{\omega_4}{\omega_3}=-\frac{N_{32}}{N_{42}}$

Elements are considered as redundant when their movements do not have any influence on the output rotational speed. Transformation of the gear graph consists especially in: neglecting of these vertices (and adjacent edges) which represent the temporary redundant gear elements.

4 Final remarks

In the paper, the transformations of graphs representing mechanical systems were considered. It was shown that in case of some exemplary mechanical artifacts e.g. a planetary gear, an automatic gear and a truss – the applied transformations of their graphs allow for an effective and an algorithmic performance of some different engineer tasks: (t1) assignment of gear functional scheme to a graph, (t2) ratio analysis and (t3) calculation of truss forces. The relatively wide reference review was done, focusing attention only on transformation aspects of graph-based models of mechanical systems. General review on graphs in mechanics can be found in [11,15]. The summary of the considered problems of modeling mechanical systems by means of graphs can be shown via a scheme presented in Fig. 4. Graphs are here considered as Artificial Intelligence tools because after knowledge transformation from mechanics to graph theory the problem can be solved automatically. After retransformation of the obtained solution from graph theory field the solution understandable for mechanical engineers is obtained.

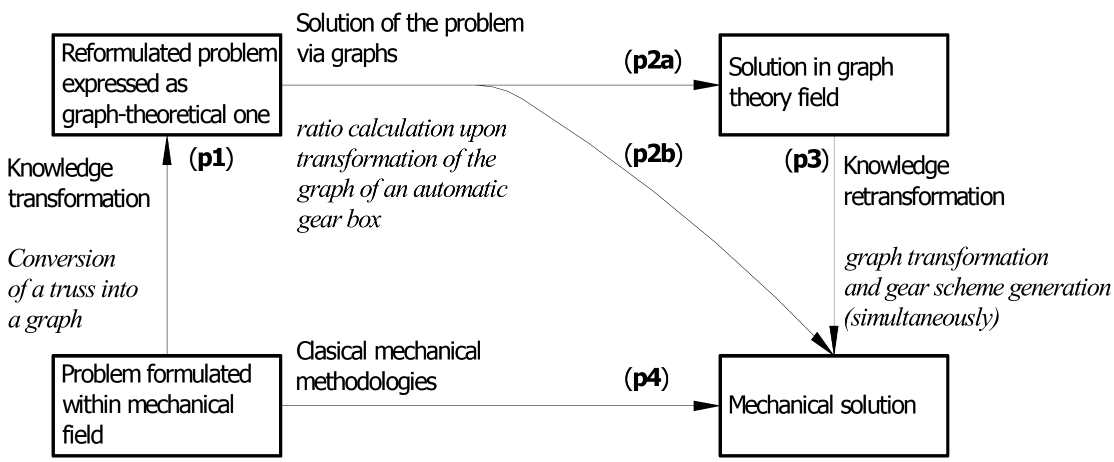


Fig.4. Scheme of usage of graph-based models of mechanical systems from AI point of view

The path (p1) in the scheme is illustrated in the paper via conversion truss-graph upon some graph transformations. The result (forces in the truss rods) is explicit so path (p2b) is used. In the case of ratio calculation we also move along the path (p2b). The path (p2a) is actively used is some tasks described in references e.g. for enumeration of designs [11]. Then retransformation is needed what was illustrated in the paper via conversion graph-functional scheme of a gear. Even wider range of mechanical tasks can be done using the graph-based models what is described in the quoted references, especially in the unique books [4,11]. Further authors' investigations are focused on usage of graph transformations in finding of the degenerate structures of gears and search for the redundant geared wheels or other redundant elements in the considered gear structures.

References

1. Borkowski A., Grabska E., Nikodem P., Strug B.: On genetic search of optimum layout skeletal structures, Progress Advances in Intelligent Computing in Engineering, VDI Verlag, No 180 , pp. 149-157, 2002.
2. Hliniak G., Strug B.: Graph grammars and evolutionary methods in graphic design, Machine Graphics & Vision, No 9, 1/2, pp. 5-13, 2000.
3. Hsu C.-H.: Graph notation for the kinematic analysis of differential gear trains, Journal of the Franklin Institute, Vol. 329 (5), pp. 859–867, 1992.
4. Kaveh A.: Structural Mechanics: Graph and Matrix Methods, Research Studies Press Ltd, Hertfordshire, 1995.
5. Li X., Schmidt L.C., He W., Li L., Qian Y.: Transformation of an EGT grammar: New grammar, new designs. ASME J. of Mech. Des., Vol. 126, No 4, pp. 753-756, 2004.
6. Rudolph S.: Semantic validation scheme for graph-based engineering design grammars; Chapter in the book - Ed. J. S. Gero: Design computing and cognition’06 (Part 7), pp. 541-560, Springer Netherlands, 2006.
7. Schmidt L. C., Shetty H., Chase S.: A graph grammar approach for structure synthesis of mechanisms, ASME Journal of Mechanical Design, Vol. 122, pp. 371-376, 2000.
8. Schmidt L. C., Cagan J.: GGREADA: A graph grammar-based machine design algorithm, Research in Engineering Design, Vol. 9, pp. 195-213, 1997.
9. Shai O.: Transforming engineering problems through graph representations, Advanced Engineering Informatics, Vol. 17, No. 2, pp. 77 - 93, April, 2003.
10. Szuba J., Schürr A., Borkowski A.: GraCAD – graph-based tool for conceptual design. Graph Transformation: Ist International Conference; Proceedings ICGT, pp.363-377, 2002.
11. Tsai L.-W.: Enumeration of kinematic structures according to function, CRC Press, Boca Raton, FL 33487, USA, 2001.
12. Tsai L-W., Magrab E.B., Mogalapalli S.N.: A CAD system for the optimization of gear ratios for automotive automatic transmissions, Department of Mechanical Engineering University of Maryland College Park, Maryland 20742. September 27, 1992.
13. Westfechel B.: Graph-based product and process management in mechanical design, RWTH Aachen, 1999.
14. Wojnarowski J., Bogucki Z.: Mechanics with exemplary calculation exercises (in Polish), Silesian University of Technology, Gliwice, 1991.
15. Wojnarowski J., Kopeć J., Zawiślak S.: Gears and graphs, Journal of Applied and Theoretical Mechanics, Vol. 44, No. 1, pp.139-162, 2006.
16. Wojnarowski J., Zawiślak S.: Modeling of mechanical system by means of matroid, Mechanism and Machine Theory, Vol. 36, No 6, pp. 717-724, 2001.
17. Zawiślak S.: Graph-based methodology as a support of conceptual design of planetary gears, Teoria maszyn i mechanizmów, Ed.: Józef Wojnarowski, Mirosław Galicki, Part 1, Oficyna Wydawnicza Uniwersytetu Zielonogórskiego, Zielona Góra, pp. 405-411, 2006.
18. Zawiślak S.: Artificial intelligence aided design of gears based on graph-theoretical models, IFToMM 2007: Twelfth World Congress in Mechanism and Machine Science, Besancon, France, June 17-21, 2007, (on line: www.iftomm2007.com).
19. Zawiślak S., Jagosz A.: An application of evolutionary algorithms and graph-based method of stress calculation for truss optimization, (in preparation) Polish Conference on Evolutionary Algorithms and Global Optimization, Warsaw TU, June 2008.