Robotics
Exercise 12

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1 Controllability

Consider the local linearization of the cart-pole,

$$\dot{x} = Ax + Bu \;, \quad A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{g}{\frac{4}{3}l - c_2} & 0 \end{pmatrix} \;, \quad B = \begin{pmatrix} 0 \\ c_1 \\ 0 \\ \frac{-c_1}{\frac{4}{3}l - c_2} \end{pmatrix}$$

Is the system controllable?

2 Stable control for the cart-pole

Consider a linear controller $u = w^\top x$ with 4 parameters $w \in \mathbb{R}^4$ for the cart-pole.

a) What is the closed-loop linear dynamics $\dot{x} = \widehat{A}x$ of the system?

b) Test if the controller with $w = (1.00000, 2.58375, 52.36463, 15.25927)$ (computed using ARE) is asymtotically stable. What are the eigenvalues?

c) Come up with a method that finds parameters w such that the closed-loop system is "maximally stable" around $x^* = (0, 0, 0, 0)$ (e.g., asymptotically stable with fastest convergence rate).

Output the optimal parameters and test them on the cart-pole simulation you developed in exercise 9 (in `course/07-cartPole`).

3 Lyapunov stability

Recall that a general controlled dynamic system can be described with the Euler-Lagrange equation as

$$\underbrace{Bu}_{\text{control}} = \frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = \underbrace{M}_{\text{inertia}}\ddot{q} + \underbrace{\dot{M}\dot{q} - \frac{\partial T}{\partial q}}_{\text{Coriolis}} + \underbrace{\frac{\partial U}{\partial q}}_{\text{gravity}}$$

Consider a dynamic *without* Coriolis forces and constant M (independent of a and $\dot{q}$).

Can you give sufficient conditions on $U(q)$ (potential energy) and $u(q)$ (control policy) such that energy is a Lyapunov function?