Robotics
Exercise 7

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1 Particle Filtering the location of a car

Start from the code in `RoboticsCourse/05-car`.

The `CarSimulator` simulates a car exactly as described on slide 03:48 (using Euler integration with step size 1sec). At each time step a control signal $u = (v, \phi)$ moves the car a bit and Gaussian noise with standard deviation $\sigma_{\text{dynamics}} = .03$ is added to x, y and θ. Then, in each step, the car meassures the relative positions to some landmars, resulting in an observation $y_t \in \mathbb{R}^{m\times 2}$; these observations are Gaussian-noisy with standard deviation $\sigma_{\text{observation}} = .5$. In the current implementation the control signal $u_t = (.1, .2)$ is fixed (roughly driving circles).

a) Odometry (dead reckoning): First write a particle filter (with $N = 100$ particles) that ignores the observations. For this you need to use the cars system dynamics (described on 03:48) to propagate each particle, and add some noise σ_{dynamics} to each particle (step 3 on slide 07:23). Draw the particles (their x, y component) into the display. Expected is that the particle cloud becomes larger and larger.

b) Next implement the likelihood weights $w_i \propto P(y_t|x_t^i) = \mathcal{N}(y_t|y(x_t^i), \sigma) \propto e^{-\frac{1}{2}(y_t - y(x_t^i))^2/\sigma^2}$ where $y(x_t^i)$ is the (ideal) observation the car would have if it were in the particle possition x_t^i. Since $\sum_i w_i = 1$, normalize the weights after this computation.

c) Test the full particle filter including the likelihood weights (step 4) and resampling (step 2). Test using a larger ($10\sigma_{\text{observation}}$) and smaller ($\sigma_{\text{observation}}/10$) variance in the computation of the likelihood.

2 Gaussians

On slide 06:11 there is the definition of a multivariate (n-dim) Gaussian distribution. Proof the following using only the definition. (You may ignore terms independent of x.)

a) Proof that:

$\mathcal{N}(x|a, A) = \mathcal{N}(a|x, A)$

$\mathcal{N}(x \,|\, a, A) = |F| \; \mathcal{N}(Fx \,|\, Fa,\; FAF^\top)$

$\mathcal{N}(Fx + f \,|\, a, A) = \frac{1}{|F|} \; \mathcal{N}(x \,|\; F^{-1}(a - f),\; F^{-1}AF^{-\top})$

b) Prove:

$\mathcal{N}(x \,|\, a, A) \; \mathcal{N}(x \,|\, b, B) \propto \mathcal{N}\big(x \;\big|\; (A^{-1} + B^{-1})^{-1}[A^{-1}a + B^{-1}b] \;,\; (A^{-1} + B^{-1})^{-1}\big)$

c) Prove:

$\int_y \mathcal{N}(x \,|\, a + Fy, A) \; \mathcal{N}(y \,|\, b, B) \; dy = \mathcal{N}(x \,|\, a + Fb, A + FBF^\top)$