Robotics

Path Optimization

very briefly

Marc Toussaint
U Stuttgart

Outline

- These are only some very brief notes on path optimization

- The aim is to explain how to *formulate* the optimization problem. Concerning the optimization algortihm itself, refer to the *Optimization* lecture.

From inverse kinematics to path costs

- Recall our optimality principle of inverse kinematics

$$\operatorname*{argmin}_q \|q - q_0\|^2_W + \|\Phi(q)\|^2$$

- A trajectory $q_{0:T}$ is a sequence of robot configurations $q_t \in \mathbb{R}^n$
- Consider the cost function

$$f(q_{0:T}) = \sum_{t=0}^T \|\Psi_t(q_{t\text{-}k}, .., q_t)\|^2 + \sum_{t=0}^T \|\Phi_t(q_t)\|^2$$

(where $(q_{-k}, .., q_{\text{-}1})$ is a given prefix)

- $\Psi_t(q_{t\text{-}k}, .., q_t)$ represents **control costs**
  k denotes the **order** of the control costs
  $\Phi_t(q_t)$ represents **task costs**
  (More generally, task costs could depend on $\Phi_t(q_{t\text{-}k}, .., q_t)$)

Control costs

- The $\Psi_t(q_{t\text{-}k}, .., q_t)$ can penalize various things:

$k = 0$	$\Psi_t(q_t) = q_t - q_0$	penalize offset from zero
$k = 1$	$\Psi_t(q_{t\text{-}1}, q_t) = q_t - q_{t\text{-}1}$	penalize velocity
$k = 2$	$\Psi_t(q_{t\text{-}2}, .., q_t) = q_t - 2q_{t\text{-}1} + q_{t\text{-}2}$	penalize acceleration
$k = 3$	$\Psi_t(q_{t\text{-}3}, .., q_t) = q_t - 3q_{t\text{-}1} + 3q_{t\text{-}2} - q_{t\text{-}3}$	penalize jerk

- The big $\Phi_t(q_t)$ imposes tasks as for inverse kinematics

Choice of optimizer

$$f(q_{0:T}) = \sum_{t=0}^{T} \|\Psi_t(q_{t\text{-}k}, .., q_t)\|^2 + \sum_{t=0}^{T} \|\Phi_t(q_t)\|^2$$

Is in the form of the so-called **Gauss-Newton** optimization problem, and can be solved using such 2nd order methods.

(Note that the pseudo Hessian is a banded, symmetric, positive-definite matrix.)

- Alternativ: formulate hard constraints in the framework of constrained optimization