Detection and Length and Orientation Measurement of Extended Targets

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Abstract: A method for rapid detection of extended targets and for measuring the length and the orientation in 2D angle space is presented. The detector and the measurements are based on the distribution of the generalized monopulse ratio. The statistical performance of the estimators is given. This method facilitates the initialization of group tracking algorithms and handling partially connected convoy targets. The performance measures may be used as a priori knowledge in tracking.

1 Introduction

Tracking groups of targets or of extended targets is a topic of special interest in radar and sonar applications. In many cases resolution into individual scatterers is not necessary and the object of interest has the shape of a line target, e.g. in ballistic missile defense or GMTI radar convoy tracking. In these cases one simply wants to estimate the centroid and the extent of this “line target”. On the signal processing level one may apply high resolution methods to resolve a complex extended target into individual scatterers. Multiple beamforming or sophisticated superresolution methods are applied requiring an array antenna with digital beamforming (multiple target Maximum-Likelihood estimation (MLE) or non-parametric methods like MUSIC). In [1] the problem of convoy tracking was solved by estimating the target length at signal processing level by multiple beams with varying separation for side-looking radar with only azimuth estimation. The extension to 2D angle estimates would require a more time consuming search procedure. On the tracking level one can study the distribution of the sequence of measurements in 3D Cartesian coordinates and fit a Gaussian mixture to these. This approach has been pursued in [2]. However, waiting for a sufficient number of measurement samples may be time consuming and often a rapid decision is desired. Besides the problem of estimating the length of an extended target there is also the detection problem. A reliable detector for extended targets has not been developed yet.

At the signal processing level a very sensitive instrument for detecting extended target is available in most modern radars, the monopulse angle estimator, [5]. There have been

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some attempts to estimate the parameters of an extended target directly from the monopulse outputs [3], [4]. Recently the distribution of the monopulse estimate for multiple targets has been described in very general terms [5], [6]. These distributions can be used to derive an estimator of the centroid and the extent by calculating the covariance matrix of the monopulse ratio. In particular, for a 2D-monopulse antenna with azimuth and elevation estimates one can obtain in this way a direct estimate the orientation of an extended target in both angles. This estimator has been derived in [7] for extended targets consisting of Swerling I and II scatterers. Additionally, a statistical description of the centroid and of the length of extended targets has been obtained. The covariance of the monopulse ratio allows to derive a detector for extended targets. Using the generalized monopulse procedure these results can be applied for arbitrary number and types of beams. It is thus applicable for planar, volume arrays or adaptive arrays.

2 Generalized Monopulse Formula and Statement of Problem

The generalized monopulse [5] determines the maximum of a "beamforming type" function $\hat{\boldsymbol{\theta}} = \arg\max_{\boldsymbol{\theta}} |S(\boldsymbol{\theta})|^2$ with $S(\boldsymbol{\theta}) = \mathbf{w}(\boldsymbol{\theta})^H \mathbf{z}$, a complex data vector $\mathbf{z}$ and a vector $\mathbf{w}$ for beamforming depending on unknown parameters $\boldsymbol{\theta} = (\theta_1, \theta_2, ...)$, typically target azimuth, elevation. The generalized monopulse formula based on an initial estimate $\boldsymbol{\theta}_0$ is, [5]:

$$\hat{\boldsymbol{\theta}} = \boldsymbol{\theta}_0 - \mathbf{C}\left(\mathrm{Re}\{\mathbf{R}_K\} - \boldsymbol{\mu}\right) \tag{1}$$

$\mathbf{C}$, $\boldsymbol{\mu}$ are correction quantities for slope and bias. $\mathbf{R}_K = \sum_{k=1}^{K} \mathbf{D}_k \bar{S}_k \,/\, \sum_{k=1}^{K} |S_k|^2$ is the averaged monopulse ratio from K beam outputs $\mathbf{B}_k = \left(\mathbf{D}_k^T, S_k\right)^T$ where $\mathbf{D}_k = \mathbf{D}(t_k)$, $S_k = S(t_k)$ denote the difference and sum beams at times t_k. The overbar denotes complex-conjugate. All beams are formed into $\boldsymbol{\theta}_0$, the antenna look direction. The key point is that this procedure can be applied to nearly any kind of beamforming (with deterministic/ adaptive weights), [5]. The vector of beam outputs $\mathbf{B}_k$ has then the structure $\mathbf{B}_k = b_k \boldsymbol{\alpha} + \mathbf{v}_k$ with $\boldsymbol{\alpha}^T = \left(\boldsymbol{\alpha}_{\mathbf{D}}^T, \alpha_S\right)$, $\boldsymbol{\alpha}_{\mathbf{D}}^T = \left(\mathbf{d}_1^H \mathbf{a}(\boldsymbol{\theta}), \ldots, \mathbf{d}_M^H \mathbf{a}(\boldsymbol{\theta})\right)$, $\alpha_S = \mathbf{w}^H \mathbf{a}(\boldsymbol{\theta})$ with $\mathbf{d}_1, \cdots \mathbf{d}_M, \mathbf{w}$ the weight vectors for difference and sum beam forming, the array plane wave model $\mathbf{a}(\boldsymbol{\theta})$ and the complex target amplitude b_k. The noise contribution is given by $\mathbf{v}_k^T = \left(\mathbf{d}_1^H \mathbf{n}_k, \ldots, \mathbf{d}_M^H \mathbf{n}_k, \mathbf{w}^H \mathbf{n}_k\right)$. Denote the covariance matrix of this beam output vector by

$$\mathrm{cov}\{\mathbf{B}\} = \mathbf{G} = \begin{bmatrix} \mathbf{G}_{\mathbf{D}} & \mathbf{G}_{\mathbf{D}S} \\ \mathbf{G}_{\mathbf{D}S}^H & G_S \end{bmatrix}. \tag{2}$$

Covariance $\mathbf{G}$ decomposes into $\mathbf{G} = \mathbf{G}_{\text{signal}} + \mathbf{G}_{\mathbf{v}}$. The noise/ interference contribution $\mathbf{v}$ is assumed i.i.d. complex Gaussian. If these K observations of a set of N Swerling I-II

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independent point scatterers with power σ_n^2 are given, then $\mathbf{B}_k = \sum_{n=1}^{N} b_k(n)\boldsymbol{\alpha}(\boldsymbol{\theta}_n) + \mathbf{v}_k$, $\mathrm{E}\left\{\left|b_k(n)\right|^2\right\} = \sigma_n^2$ and the M+1 beam outputs may be modeled as a sum of independent Gaussians with p.d.f.

$$p(\mathbf{B}_1,\ldots,\mathbf{B}_K) = \frac{\exp\left[-K\,\mathrm{tr}\left\{\mathbf{G}^{-1}\boldsymbol{\Gamma}\right\}\right]}{\left(\pi^{M+1}\left|\mathbf{G}\right|\right)^K} \tag{3}$$

and with $\boldsymbol{\Gamma} = \frac{1}{K}\sum_{k=1}^{K}\mathbf{B}_k\mathbf{B}_k^H = \begin{bmatrix}\boldsymbol{\Gamma}_{\mathbf{D}} & \boldsymbol{\Gamma}_{\mathbf{D}S} \\ \boldsymbol{\Gamma}_{\mathbf{D}S}^H & \boldsymbol{\Gamma}_S\end{bmatrix}$, $\mathbf{G} = \sum_{n=1}^{N}\sigma_n^2\boldsymbol{\alpha}(\theta_n)\boldsymbol{\alpha}(\theta_n)^H + \mathbf{G}^{\mathrm{v}}$.

We introduce the vector $\mathbf{R}(\boldsymbol{\theta}) = \left(\frac{\alpha_{d_1}(\boldsymbol{\theta})}{\alpha_s(\boldsymbol{\theta})},\ldots,\frac{\alpha_{d_M}(\boldsymbol{\theta})}{\alpha_s(\boldsymbol{\theta})}\right)^T$ of true signal monopulse ratios and consider the statistics averaged over the independent scatterers (indicated by the subscript $\boldsymbol{\theta}$) $P_s = \sum_{n=1}^{N}\sigma_n^2\left|\alpha_s(\boldsymbol{\theta}_n)\right|^2$, and $\mathbf{m}_{\mathbf{R}} = E_{\boldsymbol{\theta}}\left\{\mathbf{R}(\boldsymbol{\theta})\right\} = \sum_{n=1}^{N}\frac{\sigma_n^2\left|\alpha_s(\boldsymbol{\theta}_n)\right|^2}{P_s}\mathbf{R}(\boldsymbol{\theta}_n)$ and $\mathrm{cov}_{\boldsymbol{\theta}}\left\{\mathbf{R}(\boldsymbol{\theta})\right\} = E_{\boldsymbol{\theta}}\left\{\mathbf{R}(\boldsymbol{\theta})\mathbf{R}(\boldsymbol{\theta})^H\right\} - \mathbf{m}_{\mathbf{R}}\mathbf{m}_{\mathbf{R}}^H$. Then we have

$$\mathbf{G} = P_s\begin{bmatrix}\mathbf{m}_{\mathbf{R}}\mathbf{m}_{\mathbf{R}}^H + \mathrm{cov}_{\boldsymbol{\theta}}\left\{\mathbf{R}(\boldsymbol{\theta})\right\} & \mathbf{m}_{\mathbf{R}} \\ \mathbf{m}_{\mathbf{R}}^H & 1\end{bmatrix} + \mathbf{G}^{\mathrm{v}} \tag{4}$$

If we know the statistical features of $E_{\boldsymbol{\theta}}\left\{\mathbf{R}(\boldsymbol{\theta})\right\}$ and $\mathrm{cov}_{\boldsymbol{\theta}}\left\{\mathbf{R}(\boldsymbol{\theta})\right\}$, then we can obtain from (1) the ML estimates of $E_{\boldsymbol{\theta}}\left\{\Delta\boldsymbol{\theta}\right\}$ and $\mathrm{cov}_{\boldsymbol{\theta}}\left\{\Delta\boldsymbol{\theta}\right\}$, where $\Delta\boldsymbol{\theta} = \boldsymbol{\theta} - \boldsymbol{\theta}_0$, because

$$E_{\boldsymbol{\theta}}\left\{\Delta\boldsymbol{\theta}\right\} = -\mathbf{C}\left(E_{\boldsymbol{\theta}}\left\{\mathrm{Re}\left\{\mathbf{R}(\boldsymbol{\theta})\right\}\right\} - \boldsymbol{\mu}\right),\quad \mathrm{cov}_{\boldsymbol{\theta}}\left\{\Delta\boldsymbol{\theta}\right\} = \mathbf{C}\,\mathrm{cov}_{\boldsymbol{\theta}}\left\{\mathrm{Re}\left\{\mathbf{R}(\boldsymbol{\theta})\right\}\right\}\mathbf{C}^T \tag{5}$$

$$\text{and } \widehat{E_{\boldsymbol{\theta}}\left\{\Delta\boldsymbol{\theta}\right\}} = -\mathbf{C}\left(\widehat{E_{\boldsymbol{\theta}}\left\{\mathbf{R}(\boldsymbol{\theta})\right\}} - \boldsymbol{\mu}\right),\quad \widehat{\mathrm{cov}_{\boldsymbol{\theta}}\left\{\Delta\boldsymbol{\theta}\right\}} = \mathbf{C}\,\widehat{\mathrm{cov}_{\boldsymbol{\theta}}\left\{\mathbf{R}(\boldsymbol{\theta})\right\}}\,\mathbf{C}^T. \tag{6}$$

$\hat{\mathbf{x}}$ stands for the ML estimator of an unknown value $\mathbf{x}$ (scalar, vector or matrix).

3 Statistical Characterization of Centroid and Extent

In [7] it is shown that the ML estimates of P_s, $E_{\boldsymbol{\theta}}\left\{\mathbf{R}(\boldsymbol{\theta})\right\}$ and $\mathrm{cov}_{\boldsymbol{\theta}}\left\{\mathbf{R}(\boldsymbol{\theta})\right\}$ are

$$\begin{aligned}\widehat{\mathbf{m}_{\mathbf{R}}} = \widehat{E_{\boldsymbol{\theta}}\left\{\mathbf{R}(\boldsymbol{\theta})\right\}} &= \mathrm{Re}\left\{\frac{\boldsymbol{\Gamma}_{\mathbf{D}S} - \mathbf{G}_{\mathbf{D}S}^{\mathrm{v}}}{\Gamma_S - G_S^{\mathrm{v}}}\right\} \\ \widehat{\mathrm{cov}_{\boldsymbol{\theta}}\left\{\mathbf{R}(\boldsymbol{\theta})\right\}} &= \mathrm{Re}\left\{\frac{\boldsymbol{\Gamma}_{\mathbf{D}} - \mathbf{G}_{\mathbf{D}}^{\mathrm{v}}}{\Gamma_S - G_S^{\mathrm{v}}}\right\} - \widehat{\mathbf{m}_{\mathbf{R}}}\,\widehat{\mathbf{m}_{\mathbf{R}}}^T\end{aligned} \tag{7}$$

and $\widehat{P_s} = \Gamma_S - G_S^{\text{v}}$. The MLEs have a meaning only if $\widehat{P_s} = \Gamma_S - G_S^{\text{v}} > 0 \Leftrightarrow \|\mathbf{S}\|^2 > KG_S^{\text{v}}$, i.e. for positive sum beam power. Let us denote the event of a detection by $\Sigma = \left\{ \mathbf{S} = (S_1, \ldots, S_k) \mid \|\mathbf{S}\|^2 > \eta \right\}$ and denote P_Σ its probability (PD). For the variable $\widehat{\mathbf{m}_o} = \dfrac{\mathbf{\Gamma_{DS}} - \mathbf{G_{DS}^{\text{v}}}}{\Gamma_S - G_S^{\text{v}}}$, we have

$\operatorname{cov}\left\{ \operatorname{Re}\left\{\widehat{\mathbf{m}_o}\right\} \mid \Sigma \right\} = E\left\{ \operatorname{Re}\left\{\widehat{\mathbf{m}_o}\right\} \operatorname{Re}\left\{\widehat{\mathbf{m}_o}\right\}^T \mid \Sigma \right\} - \operatorname{Re}\left\{ E\left\{ \widehat{\mathbf{m}_o} \mid \Sigma \right\}\right\} \operatorname{Re}\left\{ E\left\{ \widehat{\mathbf{m}_o} \mid \Sigma \right\}\right\}^T$ and for every complex vector we have $\operatorname{Re}\{\mathbf{z}\}\operatorname{Re}\{\mathbf{z}\}^T = \frac{1}{2}\left[\operatorname{Re}\left\{\mathbf{z}\mathbf{z}^H\right\} + \operatorname{Re}\left\{\mathbf{z}\mathbf{z}^T\right\}\right]$. So knowledge of $E\left\{\widehat{\mathbf{m}_o} \mid \Sigma\right\}$, $E\left\{\widehat{\mathbf{m}_o}\widehat{\mathbf{m}_o}^T \mid \Sigma\right\}$ and $E\left\{\widehat{\mathbf{m}_o}\widehat{\mathbf{m}_o}^H \mid \Sigma\right\}$ provides a statistical description of $\operatorname{Re}\left\{\widehat{\mathbf{m}_\mathbf{R}}\right\}$. In [7] these expectations values are given (can be presented in the long version of the paper).

4 Detection of Extended Targets

One of our key results is the ability to estimate $\operatorname{cov}_{\boldsymbol{\theta}}\left\{\mathbf{R}(\boldsymbol{\theta})\right\}$ which leads to a detection criterion. We propose as a Multiple Signal Indicator (MSI) the test

$$\widehat{MSI} = \frac{1}{M}\operatorname{tr}\left\{\widehat{\operatorname{cov}_{\boldsymbol{\theta}}\left\{\mathbf{R}(\boldsymbol{\theta})\right\}}\right\} \underset{>}{\overset{<}{}} T_{MS} \quad \text{or} \quad \widehat{MSI} = \frac{1}{M}\operatorname{tr}\left\{\frac{\mathbf{\Gamma_D} - \mathbf{G_D^{\text{v}}}}{\Gamma_S - G_S^{\text{v}}} - \widehat{\mathbf{m}_o}\widehat{\mathbf{m}_0}^H\right\} \underset{>}{\overset{<}{}} T_{MS} \tag{8}$$

There are various reasons for choosing this criterion as explained in [7]. Basically the complex monopulse ratio covariance matrix is just the quantity which is most sensitive against non-point targets. In order to determine a suitable threshold T_{MS} we derive the first two moments $\operatorname{E}\left\{\widehat{MSI}\middle|\Sigma\right\}, \operatorname{cov}\left\{\widehat{MSI}\middle|\Sigma\right\}$. The result is

$$\operatorname{E}\left\{\widehat{MSI}\middle|\Sigma\right\} = \frac{1}{M}\operatorname{tr}\left\{\operatorname{E}\left\{\frac{\mathbf{\Gamma_D} - \mathbf{G_D^{\text{v}}}}{\Gamma_S - G_S^{\text{v}}}\middle|\Sigma\right\} - \operatorname{E}\left\{\widehat{\mathbf{m}_o}\widehat{\mathbf{m}_o}^H \middle|\Sigma\right\}\right\} \tag{9}$$

$$\begin{aligned}
&\operatorname{cov}\left\{\widehat{MSI}\middle|\Sigma\right\} = \frac{1}{M^2}\sum_{m,\mu=1}^{M}\operatorname{E}\left\{\hat{M}_{m,m}^{\mu,\mu}\middle|\Sigma\right\} - \operatorname{E}\left\{\widehat{MSI}\middle|\Sigma\right\}^2 \\
&\text{with} \quad \hat{M}_{m,l}^{\mu,t} = \left(\widehat{\operatorname{cov}_{\boldsymbol{\theta}}\left\{\mathbf{R}(\boldsymbol{\theta})\right\}}\right)_{m.l}\left(\widehat{\operatorname{cov}_{\boldsymbol{\theta}}\left\{\mathbf{R}(\boldsymbol{\theta})\right\}}\right)_{\mu,t}
\end{aligned} \tag{10}$$

The expressions for $\operatorname{E}\left\{\hat{M}_{m,l}^{\mu,t}\middle|\Sigma\right\}$, $\operatorname{E}\left\{\dfrac{\mathbf{\Gamma_D} - \mathbf{G_D^{\text{v}}}}{\Gamma_S - G_S^{\text{v}}}\middle|\Sigma\right\}$ and $\operatorname{E}\left\{\widehat{\mathbf{m}_o}\widehat{\mathbf{m}_o}^H\middle|\Sigma\right\}$ are given in [7].

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5 Numerical Performance

The performance depends of course on the shape of the sum/ difference beams, on the monopulse characteristic and the coupling between azimuth/ elevation difference beams. Therefore we use for the simulations a realistic array antenna typical for a modern multi-function radar with adaptive beamforming. A generic array of this type with 902 elements and 32 irregular subarrays has been used in previous simulations in [5], [6], [7]. Of course, we need multiple snapshots to measure an extended target, i.e. K must be greater 1. The remarkable point is that the monopulse ratio is so sensitive that a fairly small number of snapshots is sufficient.

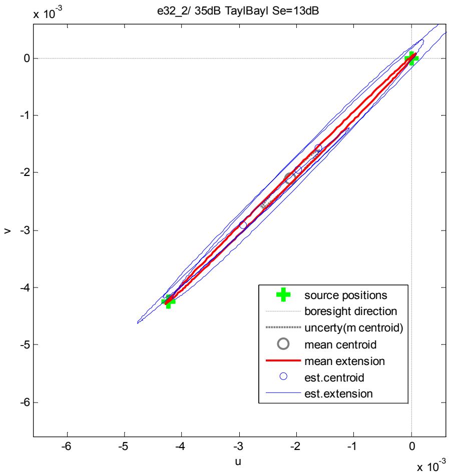

Figure 1: Centroid and extension estimation of 2 scatterers (0.1BW separation, K=10)

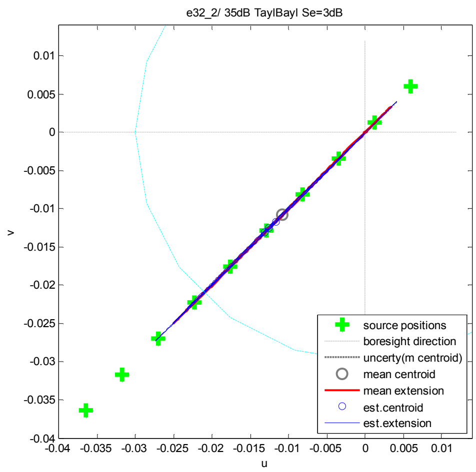

Figure 2: Estimation of centroid and extension ellipse for line target of length 1BW (10 scatterers)

Figure 1 shows results for a scenario with two targets at the position of the green crosses separated by only 1/10 of a beamwidth (BW; antenna look direction at (0,0) is indicated by the dashed line). Only K=10 data snapshots were used. The mean of the centroid and extension ellipse according to (5) are shown by bold lines and symbol; the actual ML estimates according to (7) are shown by the thin blue lines (3 realizations shown only for clarity). Note how accurate the orientation is determined.

Next we study the MSI detector of Sect. 4. We use the theoretical values for mean and variance o for calculating a threshold under Gaussian assumptions with 10% false alarms. Figure 3 shows the detection probability (PD) for this threshold setting over the separation of two point scatterers for various values of SNR. We consider very small target extensions from zero to only 1/10 BW and SNR= 6, 13, 20, 27 dB. The detector is based on 10 snapshots. Note that the extension is detected very early (0.02 BW separation at 13 dB SNR). On the other hand there is no (false) detection of a single target (value δu=0). This can be explained by the distribution of the MSI which is always positive for extended targets. Figure 4 shows the histogram of the MSI for 2 point scatterers at 0.1 BW separation (13 dB SNR, monopulse detection threshold 13 dB, 10^5

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Monte-Carlo runs). Also shown is a Gaussian fit to the given histogram and the theoretical mean and variance with corresponding Gaussian density.

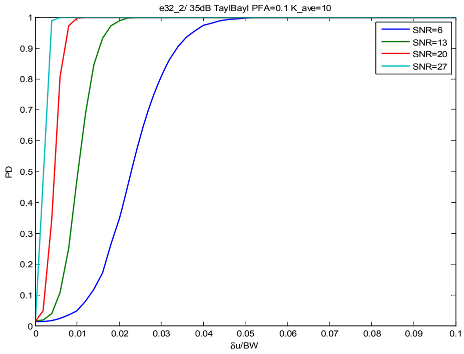

Figure 3: PD over target separation for various SNR (10% PFA, monopulse threshold 6dB, 10^4 Monte-Carlo runs for each point of curve)

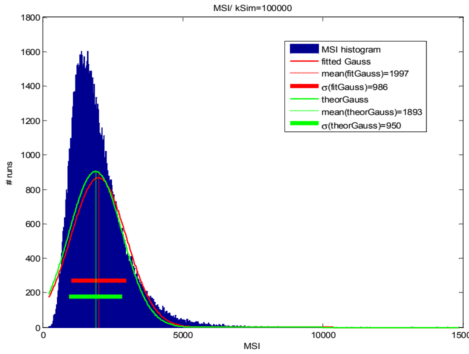

Figure 4: MSI histogram and Gaussian fit for 2 point scatterers (SNR=13dB, BW/10 separation)

Clearly the detection of extended targets becomes a problem if the extension is in the region of angular resolution. An extension ellipse will then be estimated for each antenna look direction and scatterers outside the sumbeam are attenuated by the beam pattern and will have little influence on the estimates. Therefore, extended target beyond the 3 dB beamwidth (BW) will produce a bias due to the sum beam attenuation. In Figuere 2 we have generated a scenario showing these effects: A a line of 10 scatterers (3dB SNR each) of length 1BW with a number of scatterers outside BW (indicated by the dashed circle). Although there is some bias in the estimates the orientation in 2D space is found perfectly. Targets with larger extension than BW must therefore be composed of ellipses for different beam positions. The advantage of our method is that the orientation of these ellipses is found very accurately, i.e. depends only weakly on the beam pointing. The sequence of ellipses could then be used in a Gaussian sum representation for tracking as suggested in [2].

6 Conclusions

We have presented a detector an estimator based on the complex monopulse ratio for rapid determination of the centroid and the ellipse describing extended targets which are applicable to arbitrary monopulse antennas and allow measurement for a single beam position. This is of importance for the time budget and energy management for multifunction radars. The mean and covariance of these estimators have been determined. By these the performance can be predicted for system studies and as a priori information for Bayesian tracking algorithms. The covariance of the monopulse ratio can be used to define a detector for the presence of extended targets. We have demonstrated by simulations the sensitivity of the detector and the estimators for low SNR and small target extension. Detection and estimation of the orientation of the extension ellipse may be of importance for track initialization. Also convoys in GMTI tracking which may be disconnected or merging may be handled conveniently by such sensor information.

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