

# Convoy parameter estimation with array radar

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**Abstract:** The problem of recognition of extended targets such as convoys of ground vehicles by radar is discussed. The paper focuses on target recognition during the search and track radar mode, i.e., using a real aperture mode. Radar with multi-channel array antenna is assumed. Such radar produces multi-channel echo data and offers the potential of space and space-time signal processing. Generalised spatial power estimators are used to estimate the cross-range length component of a convoy. If the radar is in motion space-time techniques have to be applied.

## 1 Introduction

We discuss the problem of recognizing an extended target such as a convoy of ground vehicles and estimation of its cross-range extension. The range component is achieved by target range measurements using appropriate resolution. The real aperture resolution of a multi-channel array antenna in conjunction with generalized spatial power estimators is used to estimate the lateral target extension. Moreover, a multi-channel array antenna offers the potential of space-time adaptive processing [K106] which is required for MTI operation in the case of moving radar. Recognition and length estimation is done during the search and tracking phase where the radar operates in real aperture mode, that means, we consider the situation where the radar is not capable of resolving the elements of the convoy. Recognition of extended targets can be utilized

1. to initiate a high resolution radar mode (e.g., SAR, ISAR) or other techniques for more detailed classification,
2. to complement measurements of the range extension component; the true target length can be obtained by vector addition of the range and the cross-range length components,
3. to support target tracking. Any knowledge about the nature of the target (point target, convoy; target length) may be suitable to improve tracking.

This paper is an extended abstract of [K111]. In [K111] the essential contents of a number of earlier conference contributions by the author and his co-authors [K107-K110] are summarized where the reader can find more detailed information.

## 2 Recognition of convoys by adaptive monopulse

The adaptive monopulse technique [Ni04, Pa00] can be used to discriminate extended targets (e.g., convoys) from point-shaped targets by an airborne multichannel array radar [Kl07]. Convoy length estimation is not considered, the output of this technique is the decision “convoy” or “point target”. Besides the parameter estimates (e.g., azimuth, elevation, radial velocity) the adaptive monopulse technique produces the standard deviations of these estimates. The standard deviations of azimuth and velocity estimates can be exploited to recognise extended targets. A point target is expected to exhibit a smaller standard deviation of the azimuth (Doppler) monopulse estimate than an extended target, particularly if the target is composed of independent scatterers as in a convoy of ground vehicles. The reason is that the monopulse technique assumes a point target and may fail whenever the target echoes are dispersed by reasons such as multipath, multi-target scenario or target extension. The decision “point” or “extended” can be taken by comparison of the standard deviations of a point target and an extended target. For more details see [Kl07].

## 3 Recognition of convoys by adaptive monopulse

A technique for recognition of convoys by estimating the cross-range length component is described. The advantage of this technique compared with the monopulse technique mentioned before is that the cross-range target length component is obtained. Hence, in conjunction with the range extension measurement, the true length of the convoy can be retrieved.

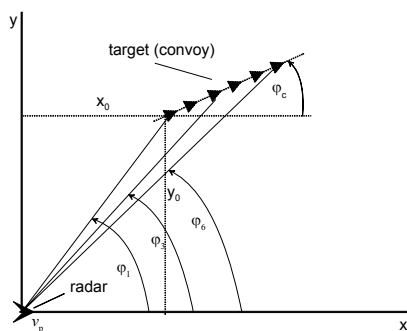


Figure 1: Radar-convoy geometry

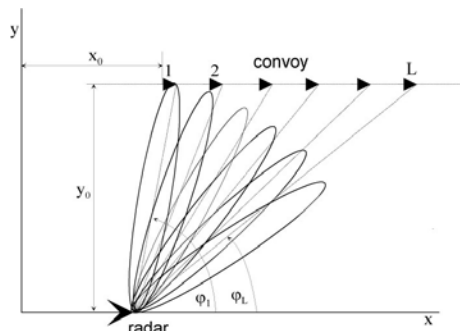


Figure 2: Multi-beam technique for convoy length estimation ( $\varphi_c=0$ ).

### 3.1 Principle of target length estimation

Figure 1 shows the scenario used in the following discussion. The radar moves at constant speed along the x-axis. The convoy consists of a number of equidistant point targets moving at constant speed on a straight path in the direction of the course angle  $\varphi_c$ . The positions of the convoy elements relative to the radar are given by the angles  $\varphi_l$ . The velocities of radar and convoy are generally different; therefore, the individual angles of the convoy elements vary with time. The  $n$ -th array element receives an echo due to the  $m$ -th pulse from the  $l$ -th convoy element

$$s(\varphi_l) = A_l \exp[j \frac{2\pi}{\lambda} \times ((x_n + vmT) \cos \varphi_l - v_c \sin \varphi_c + y_n \sin \varphi_l) \cos \vartheta_l - z_n \sin \vartheta_l)] \quad (1)$$

where  $A_l$  denotes the amplitude of the  $l$ -th arrival,  $\varphi_l$  azimuth of the  $l$ -th arrival,  $\vartheta_l$  elevation,  $x_n, y_n, z_n$  the coordinates of the  $n$ -th array element,  $m$  is the echo pulse index,  $T$  the PRI,  $v$  and  $v_c$  the velocities of radar platform and convoy. The total of signals can be comprised in a matrix

$$\mathbf{A} = (\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_L) \quad (2)$$

where the vectors  $\mathbf{a}_l$  contain the signals  $s(\varphi_l)$  due to the  $l$ -th arrival at the  $N$  array elements. The covariance of an echo signal reflected by a convoy then becomes

$$\mathbf{R} = \mathbf{A} \mathbf{C} \mathbf{A}^* \quad (3)$$

with  $\mathbf{C}$  being the correlation matrix of arrivals from individual convoy elements. Including receiver noise and interference we get

$$\mathbf{R} = \mathbf{A} \mathbf{C} \mathbf{A}^* + \mathbf{Q} \quad (4)$$

with  $\mathbf{Q}$  being the interference + noise covariance matrix. Because of the individual motion of the convoy elements and their varying aspect angles we can assume that no inter-arrival correlation occurs, i.e.,  $\mathbf{C} = \mathbf{I}$ .

### 3.2 Estimation techniques

For estimating the target length a multi-beam approach is used. Multi-beam techniques have been used for super resolution, i.e., to resolve point targets displaced by less than the conventional beam width [Wi72-Ni93]. The principle is illustrated in Figure 2. After a target has been detected a fan of beams is steered into the direction of the target. Simultaneously, a bunch of Doppler channels may be directed on the target Doppler spectrum. The total of space-time beams has the same space-time form as given in (2)

$$\mathbf{S}(\Delta\varphi) = (\mathbf{s}_1, \mathbf{s}_2, \dots, \mathbf{s}_K) \quad (5)$$

where  $\Delta\varphi$  is the angular spacing between adjacent beams  $\mathbf{s}_k, \mathbf{s}_{k+1}$  and  $A=1$ . In analogy to the signal covariance matrix (3) we can define a steering covariance matrix

$$\mathbf{B}(\Delta\varphi) = \mathbf{S}(\Delta\varphi)\mathbf{C}\mathbf{S}(\Delta\varphi)^* \quad (6)$$

In general the angular beam spacing  $\Delta\varphi$  may vary from beam to beam. In this work we assume for simplicity that the spacing is constant. Again we assume that arrivals coming from different convoy elements are mutually uncorrelated, i.e.  $\mathbf{C} = \mathbf{I}$ . Notice that the steering covariance matrix (6) is used to match a *random* signal defined by a covariance matrix (3).

The matrices  $\mathbf{R}$  (measurement) and  $\mathbf{B}(\varphi)$  (steering) represent the tools needed for determining the cross-range length of a convoy. We use generalised power estimators. They are generalised in the sense that the signal is random. Such techniques have been applied earlier to acoustic target location in shallow water [Kl80]. Numerical evaluation has shown that the maximum likelihood estimator (MLE) is best suited for the purpose of convoy length estimation. Alternative techniques [Kl08a,b] are: the Minimum Variance Estimator (MVE), the MUSIC Estimator and the Least-Squares Multi-Source Matching (LSMSM) technique [Ni93].

The MLE for a deterministic signal  $\mathbf{s}$  and noise only is given by

$$P_{\text{ML}}(\varphi) = \mathbf{b}(\varphi)^* \mathbf{s} \mathbf{s}^* \mathbf{b}(\varphi) = \text{tr}[\mathbf{s} \mathbf{s}^* \mathbf{b}(\varphi) \mathbf{b}^*(\varphi)] \quad (7)$$

which is the classical beam former for a deterministic signal. This form can be generalized to random signals by replacing the dyadic  $\mathbf{b}(\varphi) \mathbf{b}^*(\varphi)$  by the steering covariance matrix (6) and the dyadic  $\mathbf{s} \mathbf{s}^*$  by the signal + noise covariance matrix  $\mathbf{R}$ :

$$P_{\text{ML}}(\Delta\varphi) = \text{tr}[\mathbf{R} \mathbf{B}(\Delta\varphi)] \quad (8)$$

where  $\mathbf{R}$  is the signal+noise covariance matrix,  $\mathbf{B}(\Delta\varphi)$  the steering covariance matrix and  $\Delta\varphi$  the angular spacing of the beams. Including clutter and interference one gets

$$P_{\text{ML}}(\Delta\varphi) = \frac{\text{tr}(\mathbf{Q}^{-1} \mathbf{B}(\Delta\varphi) \mathbf{Q}^{-1} \mathbf{R})}{\text{tr}(\mathbf{Q}^{-1} \mathbf{B}(\Delta\varphi))} \quad (9)$$

Estimation takes place by varying  $\Delta\varphi$  until  $P_{\text{ML}}(\Delta\varphi)$  becomes maximum. In practice  $\mathbf{R}$  has to be replaced by  $\hat{\mathbf{R}}$  which is an estimate of the signal+clutter+noise covariance matrix.  $\mathbf{Q}^{-1}$  is an adaptive anti-clutter or anti-interference filter which becomes  $\mathbf{Q} = \mathbf{I}$  if the interference is white noise only. In practice  $\mathbf{Q}$  has to be estimated from secondary data with the cell under test being excluded.

During earlier studies [Kl08a,Kl10] some insight into the properties of the convoy length estimation technique was obtained:

- The MLE (9) appears to be the most appropriate technique for estimating the cross-range convoy length.
- So far *spatial* processing was considered (forming beams according to Figure 2 and maximize  $P_{ML}$  with respect to angular beam spacing). Equivalently one may perform this procedure in the time domain, i.e., form a bunch of Doppler channels and maximize  $P_{ML}$  with respect to Doppler channel spacing. This approach may be attractive because it is easier and more cost efficient to design a large *temporal* aperture (number of pulses) than a large array.
- For moving radar *space-time* clutter suppression techniques have to be involved which means using both a *temporal* and a *spatial* aperture. Then the ML estimator has to be formulated in space-time notation according to (2). It should be noted that by using both temporal and spatial aperture of equal size the resolution capability and, hence, the capability of length estimation is not improved. The larger aperture of spatial or temporal processing limits the resolution capability.

## 4 Results

Some numerical examples are presented to illustrate some properties of the convoy length measurement technique.

### 4.1 Impact of the array aperture and SNR

The resolution capability of an array antenna depends on the array size projected into the look direction. During tracking the look direction of the array varies continuously with the changing radar-target configuration. In the following examples a linear sparse array (spatially under sampled) was assumed in order to reduce computation time. A linear array in side looking geometry (array aligned with flight path) was assumed.

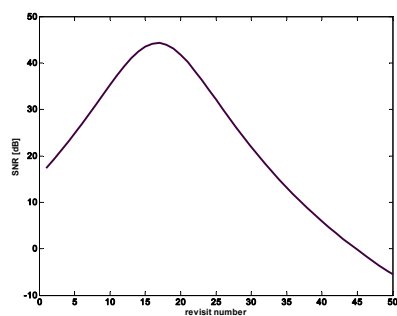


Figure 3: SNR [dB] vs. revisit number (no clutter, revisit interval 16s)

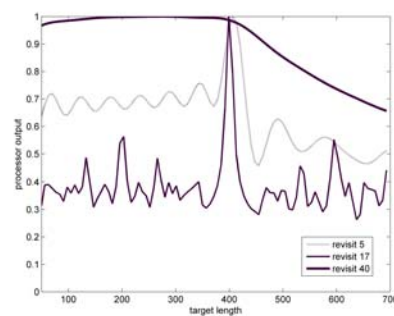


Figure 4: Cross-range target length estimation by generalised MLE, no clutter

Figure 4 shows an example for the optimization of  $P_{ML}$  (9). A clutter-free scenario was assumed. The SNR at the receiver output versus the revisit number is shown in Figure 3. At revisit 17 the radar carrier overtakes the convoy; this is the point of closest approach.

The three curves have been plotted for 3 different revisits, (No. 5, 17 and 40). The three radar-target configurations have the following properties:

1. At revisit number 17 the radar overtakes the convoy. The SNR is maximal (see Figure 3) and, due to the geometry of a side looking array; the target is at broadside so that the projected array aperture is maximal. These conditions are most favourable for convoy length estimation.
2. At revisit 5 the SNR is more than 20 dB lower than at revisit 17. Moreover, the projected array aperture is much smaller because the target appears at a positive angle different from  $90^\circ$  (broadside).
3. After the radar has overtaken the convoy (revisit  $> 17$ , here: 40) we face a similar situation as at revisit 5 (see 2.), characterized by reduced SNR and reduced projected aperture.

These properties are reflected in the optimization curves shown in Figure 4. At revisit 17 the  $P_{ML}$  curve shows a very clear peak at target length 400m which is the true convoy length. At revisit 5 a peak close to the true target length is obtained. Notice the increased side lobe level. Finally, at revisit 40, no distinct maximum can be obtained, that means, no length estimation is possible.

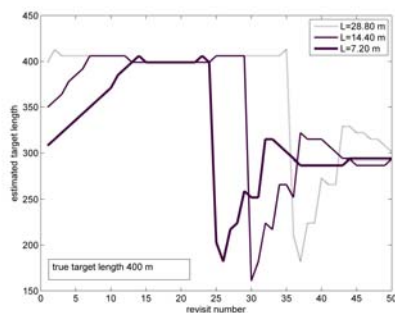


Figure 5: Estimation of the cross-range convoy length during tracking: impact of the array length L.

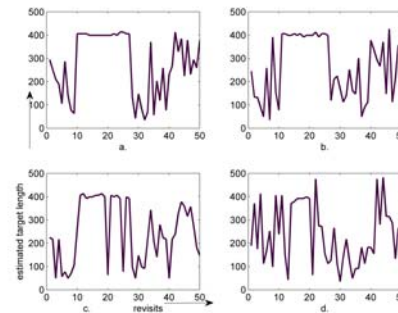


Figure 6: Impact of the sample size (a. 20; b. 10.; c. 5; d. 1), array size 14.40 m, 400 m target

Figure 5 shows examples for target length estimation during tracking. The impact of the array size and the array-target geometry can be seen clearly. The three curves have been plotted for different array size (28,80m; 14,40m; 7,20m). Each curve has a flat part roughly centered about the point of closest approach (revisit 17), also the broadside direction. In these flat areas the array is capable of estimating the target length (400m) correctly. The larger the array is the wider is the usable domain. The usable areas are limited on both sides by cut-offs beyond which no estimation is possible. Outside the flat areas the associated estimation curves run like the one at revisit 40 in Figure 4.

## 4.2 Sample size

In the preceding examples it was assumed that the covariance matrix of signal, clutter and noise  $\mathbf{R}$  in (9) is known. In practice this matrix has to be estimated from received data by averaging data vector dyadics. Assuming that the target is contained in a single range increment no data are available from auxiliary range gates. Instead, data vectors have to be taken from successive time segments. The total time, however, should not exceed a certain limit so as to prevent noticeable change of the scenario during averaging. In other words, the number of samples should be small.

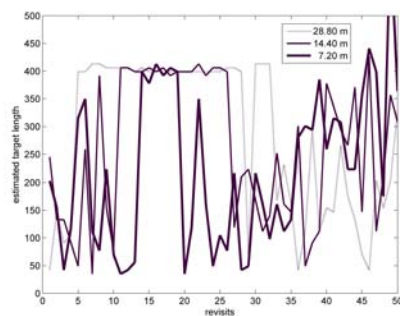


Fig. 7: Impact of the array size, 10 samples, (true target length 400m)

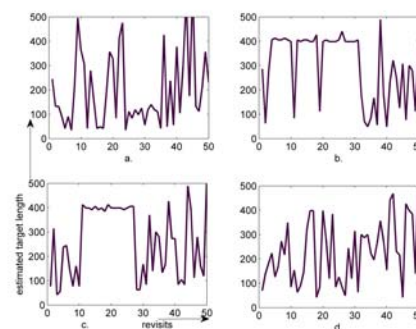


Figure 8: Clutter, 10 data samples: a. 28.80 m array, no clutter filter; b. 28.80 m array, with STAP; c. 14.40 m array, with STAP; d. 7.20 m array, with STAP (true target length 400m)

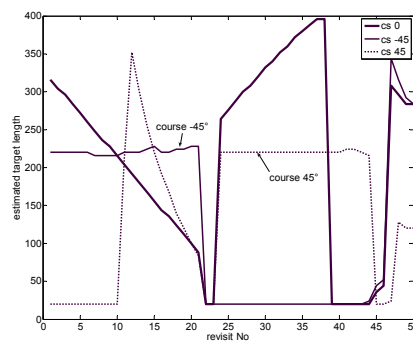


Figure 9: Tracking a convoy on a bended path (90° left turn. Left wing of the convoy path: -45°, right wing: +45°; turn at about revisit 23; true convoy length 235m.

Figure 6 gives an impression of the role of the sample size of the signal+ noise covariance matrix  $\mathbf{R}$  (no clutter). The number of samples varies from 20 to 1. The number of array elements was 96, therefore, even with 20 samples; the matrix estimate is not regular. However, one can notice that the flat area of correct length estimation is visible for 20 and 10 samples. Even for 5 samples the plateau is still visible, with a few breakdowns in between.

Looking at Figure 7 it is interesting to note that for a constant number of samples (10 in this example) the plateau usable for target length estimation widens with increasing array aperture although the dimension of the associated covariance matrix  $\mathbf{R}$  is getting larger and the number of samples is constant. This is in contrast with techniques such as adaptive jammer nulling or STAP which involve the *inverse* of a covariance matrix where the number of samples should be at least equal to the matrix dimension order to achieve a regular matrix.

### 4.3 Including ground clutter

In the previous examples we assumed that no clutter is present. This means that  $\mathbf{Q}$  in (9) is the identity matrix of receiver noise only. If clutter is included  $\mathbf{Q}$  becomes the space-time clutter+noise covariance matrix and  $\mathbf{Q}^{-1}$  the adaptive space-time clutter filter. Figure 8 shows some numerical results. The signal+clutter+noise covariance matrix has been estimated from 10 data vector samples while the inverse of the clutter covariance matrix was assumed to be perfectly known.

In Fig. 8a no clutter filter was used ( $\mathbf{Q}^{-1}=\mathbf{I}$ ). We get the expected result that no length estimation is possible. In Figs. 8b-d the length estimate has been plotted for different array size. Comparing Figure 9b with 8c we can conclude that length estimation is not affected by the clutter environment if a perfect clutter filter is applied. As in Figure 7 the range where good estimates can be achieved varies with the array aperture.

### 4.4 Role of the course angle

So far we assumed that the course angle of the convoy  $\varphi_c$  (see Figure 1) is known. Normally the course angle is unknown. Then a mismatch between the course angle  $\varphi_c$  in the signal covariance matrix  $\mathbf{R}$  and the steering course angle  $\varphi_{cs}$  in the steering covariance matrix  $\mathbf{B}$  may occur in (9):

$$P_{ML}(\Delta\varphi) = \frac{\text{tr}(\mathbf{Q}^{-1}\mathbf{B}(\Delta\varphi, \varphi_{cs})\mathbf{Q}^{-1}\mathbf{R}(\varphi_c))}{\text{tr}(\mathbf{Q}^{-1}\mathbf{B}(\Delta\varphi, \varphi_{cs}))} \quad (13)$$

Some details about the role of the course angle in convoy length estimation can be found in [K1107]. In Figure 9 it is shown how mismatch between processor ( $\varphi_{cs}$ ) and the measured data ( $\varphi_c$ ) can affect convoy length estimation. We consider a scenario where the convoy moves first in the direction  $-45^\circ$ , changes direction at revisit 22 and continues in the direction  $+45^\circ$ . Three cases with 3 different steering course angles ( $\varphi_{cs} = 0^\circ$ ;  $\varphi_{cs} = +45^\circ$ ;  $\varphi_{cs} = -45^\circ$ ) are shown:

- $\varphi_{cs} = 0$  (fat): The processor design is based on the assumption that the convoy moves on a straight path parallel to the radar path. As can be seen from the curve in Figure 9 the two wings of the convoy path are reflected clearly in the length estimation result. As long as the convoy moves in the direction  $-45^\circ$  ( $<$  revisit 23) the length measurement shows a down-slope. For  $+45^\circ$  ( $>$  revisit 23) one obtains an up-slope. Now the true course angles can be estimated by variation of the steering course angle  $\varphi_{cs}$ . The slopes give an indication in which direction the steering course angle has to be varied in order to match one of the wings of the convoy path.

- $\varphi_{cs} = -45^\circ$  (thin): Now the processor is matched to the left wing of the convoy path. As can be seen the length measurement of the left wing ( $<$  revisit 22) response shows a flat plateau close to the true convoy length. For revisits  $> 24$  ( $\varphi_c = +45^\circ$ ) the processor is completely mismatched to the convoy path so that length measurement is not possible.
- $\varphi_{cs} = +45^\circ$  (dotted): Now the processor is matched to the right wing of the convoy path ( $>$  revisit 24). Again one gets a flat plateau at the true convoy length while for revisit numbers  $< 22$  no useful estimates are obtained due to processor mismatch.

In the transient phase between course angles  $-45^\circ$  and  $+45^\circ$  (revisits 22-24) the convoy just turns left by  $90^\circ$ , i.e., some vehicles move in the direction  $-45^\circ$ , the others towards  $+45^\circ$ . Because of two different directions of motion of the two parts of the convoy we obtain 2 different Doppler frequencies which cannot be handled reasonably by the conventional Doppler processor. Therefore, no useful length estimates are obtained as long as the convoy passes the curve.

Application of the described method runs as follows. Suppose the scenario is a road consisting of straight pieces with different course angles. While tracking the convoy the steering course angle  $\varphi_{cs}$  has to be varied over all angles of interest. Applying the estimation procedure will then result in a curve consisting of horizontal pieces like those in Figure 9, indicating the convoy length. The steering course angles associated with each of the horizontal pieces indicate the course angles of the road pieces.

## 5 Summary and conclusions

This paper is a short version of [K111] where a series of conference papers by the author and his co-authors are summarized. The problem of convoy recognition including the estimation of the cross-range length component has been presented.

The estimation technique is based on generalized power estimators matched to a random signal. The procedure described may be applied in the space or Doppler domain (or both if the radar platform is moving). Notice that it is easier to generate a coherent pulse sequence than building a large array antenna.

The following results have been obtained: 1. The capability of cross-range target length estimation depends on the aperture of the array antenna. Relatively large antenna size is required to get satisfactory results. Such arrays may be realised as conformal array along the fuselage of an aircraft; 2. Estimation of the signal covariance matrix requires a relatively small number of data vector samples (smaller than number of array elements); 3. Ground clutter can be suppressed by including a clutter filter in the generalised power estimator; 4. The course angle of the convoy can be estimated during tracking.

Several open questions need further attention: 1. Can the techniques shown be applied if the convoy moves on a curved or snaky course? 2. Can the computational load be reduced by applying the generalised power estimator in a subspace with reduced dimension?

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