On the Computation of Ranking Functions for Default Rules – A Challenge for Constraint Programming

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Abstract: Qualitative conditionals of the form *If A then normally B* can be viewed as default rules, and they require a semantical treatment going beyond the models used in classical logic. Ranking functions assigning degrees of plausibility to each possible world have been proposed as an appropriate semantic formalism. In this paper, we discuss the computation of c-representations corresponding to particular ranking functions for a set $\mathcal{R}$ of qualitative conditionals. As a challenge for constraint programming, we formulate a constraint satisfaction problem $CR(\mathcal{R})$ as a declarative specification of all c-representations for $\mathcal{R}$, and we argue that employing constraint programming techniques will be advantageous for computing all minimal solutions of $CR(\mathcal{R})$.

1 Introduction

Knowledge in every-day life, in many scientific and technical disciplines, in textbooks, or in knowledge-based systems is very often expressed in the form of *if-then* rules like *If A then (normally) B*. Such a conditional expresses that there is a plausible relationship between *A* and *B*, i.e. if *A* is the case then it is plausible to assume that also *B* is the case. To give a concrete example, let *A* stand for *The car does not start* and let *B* stand for *The battery is flat*. If we observe that the car does not start, it is plausible to assume that the battery is flat.

However, assigning a truth value to *If the car does not start then normally the battery is flat*, is not obvious at all; indeed such a truth value does not make sense for instance in cases where the car does start. Instead, we say that a rational agent accepts the conditional *If the car does not start then normally the battery is flat*, if the agent deems a world where *"The car does not start and the battery is flat"* is true less suprising than a world where *"The car does not start and the battery is not flat"* is true. Note that this is fundamentally different from saying that from *the car does not start* it necessarily follows that *the battery is flat*, since the agent's belief about possible worlds still allows for exceptions, i.e. for a possible world where *"The car does not start and the battery is not flat"* is true.

In this paper, we deal with semantical approaches for conditionals that can be viewed as default rules as illustrated above. These approaches employ so-called ranking functions that order possible worlds according to their degree of surprise or their degree of

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plausibility resp. implausibility (cf. [Spo88, GMP93, GP96]). In [KI01, KI02] a criterion when a ranking function respects the conditional structure of a set $\mathcal{R}$ of conditionals is defined, leading to the notion of c-representation for $\mathcal{R}$, and it is argued that ranking functions defined by c-representations are of particular interest for model-based inference. In [BKIK08] a system that computes a c-representation for any such $\mathcal{R}$ that is consistent is described, but this c-representation may not be minimal. While the problem of finding a minimal ranking function for $\mathcal{R}$ involves an exponential search space, [Bou99] presents an algorithm for computing a minimal ranking function, but this algorithm fails to find all minimal ranking functions if there is more than one minimal one. In [Mül04] an extension of that algorithm being able to compute all minimal c-representations for $\mathcal{R}$ is presented. The algorithm developed in [Mül04] uses a non-declarative approach and is implemented in an imperative programming language.

The aim of the present paper is three-fold: First, we will present the problem of specifying all c-representations for $\mathcal{R}$ and formalize it explicitly as a high-level, problem-oriented constraint satisfaction problem $CR(\mathcal{R})$. Second, since solving $CR(\mathcal{R})$ efficiently and finding all its minimal solutions is still a difficult task, we present $CR(\mathcal{R})$ as a challenge for constraint programming. Finally, we argue that using a declarative approach based on constraint programming will be advantageous with respect to the flexibility of adding or modifying constraints, e.g. in order to further reduce the number of minimal solutions which is a topic of ongoing research.

This paper is organized as follows: In Section 2, we recall the formal background of conditional logics as it is given in [BKI08] and as far as it is needed here. In Section 3, we present an illustrative example for a conditional knowledge base and for what can be inferred from it. In Section 4, the notion of c-representation [KI02] for a set $\mathcal{R}$ of conditionals is given, and the constraint satisfaction problem $CR(\mathcal{R})$ whose solution set denotes all c-representations for $\mathcal{R}$ is defined and illustrated in Section 5. Section 6 concludes the paper and points out further work.

2 Background

We start with a propositional language $\mathcal{L}$, generated by a finite set Σ of atoms $a, b, c, \ldots$. The formulas of $\mathcal{L}$ will be denoted by uppercase Roman letters $A, B, C, \ldots$. For conciseness of notation, we will omit the logical *and*-connective, writing AB instead of $A \wedge B$, and overlining formulas will indicate negation, i.e. $\overline{A}$ means $\neg A$. Let Ω denote the set of possible worlds over $\mathcal{L}$; Ω will be taken here simply as the set of all propositional interpretations over $\mathcal{L}$ and can be identified with the set of all complete conjunctions over Σ. For $\omega \in \Omega$, $\omega \models A$ means that the propositional formula $A \in \mathcal{L}$ holds in the possible world ω.

By introducing a new binary operator $|$, we obtain the set

$$(\mathcal{L} \mid \mathcal{L}) = \{(B|A) \mid A, B \in \mathcal{L}\}$$

of *conditionals* over $\mathcal{L}$. $(B|A)$ formalizes "*if* A *then (normally)* B" and establishes a plausible, probable, possible etc connection between the *antecedent* A and the *consequence* B.

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Here, conditionals are supposed not to be nested, that is, antecedent and consequent of a conditional will be propositional formulas.

A conditional $(B|A)$ is an object of a three-valued nature, partitioning the set of worlds Ω in three parts: those worlds satisfying AB, thus *verifying* the conditional, those worlds satisfying $A\overline{B}$, thus *falsifying* the conditional, and those worlds not fulfilling the premise A and so which the conditional may not be applied to at all. This allows us to represent $(B|A)$ as a *generalized indicator function* going back to [DeF74] (where u stands for *unknown* or *indeterminate*):

$$(B|A)(\omega) = \begin{cases} 1 & \text{if } \omega \models AB \\ 0 & \text{if } \omega \models A\overline{B} \\ u & \text{if } \omega \models \overline{A} \end{cases}$$

To give appropriate semantics to conditionals, they are usually considered within richer structures such as *epistemic states*. Besides certain (logical) knowledge, epistemic states also allow the representation of preferences, beliefs, assumptions of an intelligent agent. Basically, an epistemic state allows one to compare formulas or worlds with respect to plausibility, possibility, necessity, probability, etc.

Well-known qualitative, ordinal approaches to represent epistemic states are Spohn's *ordinal conditional functions, OCFs*, (also called *ranking functions*) [Spo88], and *possibility distributions* [BDP92], assigning degrees of plausibility, or of possibility, respectively, to formulas and possible worlds. In such qualitative frameworks, a conditional $(B|A)$ is valid (or *accepted*), if its confirmation, AB, is more plausible, possible, etc. than its refutation, $A\overline{B}$; a suitable degree of acceptance is calculated from the degrees associated with AB and $A\overline{B}$.

In this paper, we consider Spohn's OCFs [Spo88]. An OCF is a function

$$\kappa : \Omega \to \mathbb{N}$$

expressing degrees of plausibility of propositional formulas where a higher degree denotes "less plausible" or "more suprising". At least one world must be regarded as being normal; therefore, $\kappa(\omega) = 0$ for at least one $\omega \in \Omega$. Each such ranking function can be taken as the representation of a full epistemic state of an agent. Each such κ uniquely extends to a function (also denoted by κ) mapping sentences and rules to $\mathbb{N} \cup \{\infty\}$ and being defined by

$$\kappa(A) \quad = \quad \begin{cases} \min\{\kappa(\omega) \mid \omega \models A\} & \text{if } A \text{ is satisfiable} \\ \infty & \text{otherwise} \end{cases} \tag{1}$$

for sentences $A \in \mathcal{L}$ and by

$$\kappa((B|A)) \quad = \quad \begin{cases} \kappa(AB) - \kappa(A) & \text{if } \kappa(A) \neq \infty \\ \infty & \text{otherwise} \end{cases} \tag{2}$$

for conditionals $(B|A) \in (\mathcal{L} \mid \mathcal{L})$. Note that $\kappa((B|A)) \geqslant 0$ since any ω satisfying AB also satisfies A and therefore

$$\kappa(A) = \min_{\omega \models A} \kappa(\omega) \leqslant \min_{\omega \models AB} \kappa(\omega) = \kappa(AB),$$

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ensuring that $\kappa(AB) - \kappa(A) \geqslant 0$.

The belief of an agent being in epistemic state κ with respect to a default rule $(B|A)$ is determined by the satisfaction relation $\models_{\mathcal{O}}$ defined by:

$$\kappa \models_{\mathcal{O}} (B|A) \quad \text{iff} \quad \kappa(AB) \; < \; \kappa(A\overline{B}) \tag{3}$$

Thus, $(B|A)$ is believed in κ iff the rank of AB (verifying the unquantified conditional) is strictly smaller than the rank of $A\overline{B}$ (falsifying the unquantified conditional). We say that κ *accepts* the conditional $(B|A)$ iff $\kappa \models_{\mathcal{O}} (B|A)$.

3 Example

We will illustrate the concepts presented in the previous section with a simple example. Suppose we want to formalize the following default rules:

- Sea animals have gills.
- Sea animals are not mammals.
- Dolphins are sea animals.
- Dolphins are mammals.

Example 1 Using the propositional variables {s, g, m, d} for sea animal, gills, mammal, and dolphin we get the knowledge base $\mathcal{R} = \{R_1, \ldots, R_4\}$ with

$$\begin{aligned} R_1&: \ (g|s) \\ R_2&: \ (\overline{m}|s) \\ R_3&: \ (s|d) \\ R_4&: \ (m|d) \end{aligned}$$

■

Figure 1 shows a ranking function κ that accepts all conditional given in $\mathcal{R}$ (this ranking function has been computed using the CONDOR@AsmL system [BKIK08]). Thus, for any $i \in \{1, 2, 3, 4\}$ it holds that $\kappa \models_{\mathcal{O}} R_i$.

For the conditional $(g|d)$ that is not contained in $\mathcal{R}$, we get $\kappa(dg) = 1$ and $\kappa(d\overline{g}) = 2$ and therefore $\kappa \models_{\mathcal{O}} (g|d)$.

On the other hand, for the conditional $(d|s)$ that is also not contained in $\mathcal{R}$, we get $\kappa(sd) = 1$ and $\kappa(s\overline{d}) = 0$ and therefore $\kappa \not\models_{\mathcal{O}} (d|s)$ so that the conditional $(d|s)$ is not accepted by κ.

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ω	$\kappa(\omega)$
$s\,g\,m\,d$	1
$s\,g\,m\,\overline{d}$	1
$s\,g\,\overline{m}\,d$	2
$s\,g\,\overline{m}\,\overline{d}$	0
$s\,\overline{g}\,m\,d$	2
$s\,\overline{g}\,m\,\overline{d}$	2
$s\,\overline{g}\,\overline{m}\,d$	3
$s\,\overline{g}\,\overline{m}\,\overline{d}$	1
$\overline{s}\,g\,m\,d$	2
$\overline{s}\,g\,m\,\overline{d}$	0
$\overline{s}\,g\,\overline{m}\,d$	4
$\overline{s}\,g\,\overline{m}\,\overline{d}$	0
$\overline{s}\,\overline{g}\,m\,d$	2
$\overline{s}\,\overline{g}\,m\,\overline{d}$	0
$\overline{s}\,\overline{g}\,\overline{m}\,d$	4
$\overline{s}\,\overline{g}\,\overline{m}\,\overline{d}$	0

Figure 1: Ranking function κ accepting the rule set $\mathcal{R}$ given in Example 1

4 C-Representations and Ranking Functions

Given a set $\mathcal{R} = \{R_1, \ldots, R_n\}$ of conditionals, a ranking function κ that accepts every R_i repesents an epistemic state of an agent accepting $\mathcal{R}$. If there is no κ that accepts every R_i then $\mathcal{R}$ is *inconsistent*. For the rest of this paper we assume that $\mathcal{R}$ is consistent.

For any consistent $\mathcal{R}$ there may be many different κ accepting $\mathcal{R}$, each representing a complete set of beliefs with respect to every possible formula A and every conditional $(B|A)$. Thus, every such κ inductively completes the knowledge given by $\mathcal{R}$, and it is a vital question whether some κ' is to be preferred to some other κ'', or whether there is a unique “best” κ. Different ways of determining a ranking function are given by *system Z* [GMP93, GP96] or its more sophistictaed extension *system* Z* [GMP93], see also [BP99]; for an approach using rational world rankings see [Wey98].

For quantitative knowledge bases of the form $\mathcal{R}_x = \{(B_1|A_1)[x_1], \ldots, (B_n|A_n)[x_n]\}$ with probability values x_i and with models being probability distributions P satisfying a probabilistic conditional $(B_i|A_i)[x_i]$ iff $P(B_i|A_i) = x_i$, a unique model can be choosen by employing the principle of maximum entropy [Par94, PV97, KI98]; the maximum entropy model is a best model in the sense that it is the most unbiased one among all models satisfying $\mathcal{R}_x$.

Using the maximum entropy idea, in [KI02] a generalization of system Z* is suggested. Based on an algebraic treatment of conditionals, the notion of *conditional indifference* of κ with respect to $\mathcal{R}$ is defined and the following criterion for conditional indifference is

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given: An OCF κ is indifferent with respect to

$$\mathcal{R} = \{(B_1|A_1), \ldots, (B_n|A_n)\}$$

iff

$$\kappa(A_i) < \infty$$

for all $i \in \{1, \ldots, n\}$ and there are rational numbers $\kappa_0, \kappa_i^+, \kappa_i^- \in \mathbb{Q},\ 1 \leqslant i \leqslant n$, such that for all $\omega \in \Omega$,

$$\kappa(\omega) = \kappa_0 + \sum_{\substack{1 \leqslant i \leqslant n \\ \omega \models A_i B_i}} \kappa_i^+ + \sum_{\substack{1 \leqslant i \leqslant n \\ \omega \models A_i \overline{B_i}}} \kappa_i^- . \tag{4}$$

Note that although $\kappa_0, \kappa_i^+, \kappa_i^-$ may be rational numbers, the world rankings $\kappa(\omega)$ are still assumed to be natural numbers. Thus finding a κ that is indifferent with respect to $\mathcal{R}$ amounts to choosing $\kappa_0, \kappa_i^+, \kappa_i^-$ such that the constraints given by (4) are satisfied. When starting with an epistemic state of complete ignorance (i.e., each world ω has rank 0), for each rule $(B_i|A_i)$ the values κ_i^+, κ_i^- determine how the rank of each satisfying world and of each falsifying world, respectively, should be changed. κ_0 is a normalization constant ensuring that there is smallest world rank 0.

While (4) allows for multiple ways of adjusting a world rank, [KI02] points out two ways to simplify the form of κ. Employing the postulate that the ranks of a satisfying world should not be changed yields the constraints

$$\kappa_i^+ = 0, \tag{5}$$

and requiring that changing the rank of a falsifying world may not result in an increase of the world's plausibility yields the constraints

$$\kappa_i^- \geqslant 0 \tag{6}$$

for all $i \in \{1, \ldots, n\}$. Requiring that κ accepts $\mathcal{R}$ can be expressed by the constraints

$$\kappa(A_i B_i) < \kappa(A_i \overline{B_i}) \tag{7}$$

again for all $i \in \{1, \ldots, n\}$. Note that (7) necessarily implies $\kappa(A_i) < \infty$.

Furthermore, under conditions (5) and (6), we have that

$$\kappa_0 = 0 \tag{8}$$

since $\mathcal{R}$ is assumed to be consistent [KI02].

Whereas in general, c-representations do not require non-negative values for κ_i^- and trivial zero-values for κ_i^+ as in (5) and (6), for the rest of this paper we will be interested only in special c-representations arising from these restrictions (cf. [KI02]) and that we will just also call c-representation.

Definition 2 Let $\mathcal{R} = \{(B_1|A_1), \ldots, (B_n|A_n)\}$. Any ranking function κ satisfying (4), (5), (6), (7), and (8) is called a *(special) c-representation* of $\mathcal{R}$.

Thus, finding a c-representation for $\mathcal{R}$ now amounts to choosing appropriate values κ_1^-, $\ldots, \kappa_n^-$.

5 The Constraint Satisfaction Problem $CR(\mathcal{R})$

For any set $\mathcal{R}$ of conditionals, we will now explicitly formulate the constraint satisfaction problem $CR(\mathcal{R})$ whose solutions are vectors of the form $(\kappa_1^-, \ldots, \kappa_n^-)$ determining c-representations of $\mathcal{R}$.

Note that using (1), the constraint (7) is equivalent to

$$\min_{\omega \models A_i \overline{B_i}} \kappa(\omega) - \min_{\omega \models A_i B_i} \kappa(\omega) > 0 \tag{9}$$

when setting $\min(\emptyset) = \infty$. We can now substitute the expression given for $\kappa(\omega)$ in (4) into (9); doing so and rearranging and simplyfing expressions (see [KI02, KI01] for details) transforms (9) into the constraint

$$\kappa_i^- > \min_{\omega \models A_i B_i} \sum_{\substack{j \neq i \\ \omega \models A_j \overline{B_j}}} \kappa_j^- - \min_{\omega \models A_i \overline{B}_i} \sum_{\substack{j \neq i \\ \omega \models A_j \overline{B_j}}} \kappa_j^- \tag{10}$$

for any $i \in \{1, \ldots, n\}$. Furthermore, using (5) and (8), we can simplify (4) by eliminating κ_0 and the sum over the κ_i^+, yielding:

$$\kappa(\omega) = \sum_{\substack{1 \leqslant i \leqslant n \\ \omega \models A_i \overline{B_i}}} \kappa_i^- \tag{11}$$

In the following, we will consider only solutions with κ_i^- being natural numbers (and not just rational numbers).

Definition 3 [$CR(\mathcal{R})$] Let $\mathcal{R} = \{(B_1|A_1), \ldots, (B_n|A_n)\}$. The constraint satisfaction problem for c-representations of $\mathcal{R}$, denoted by $CR(\mathcal{R})$, is given by the conjunction of the constraints (6) and (10) for all $i \in \{1, \ldots, n\}$.

A solution of $CR(\mathcal{R})$ is an n-tupel

$$(\kappa_1^-, \ldots, \kappa_n^-)$$

of natural numbers, and with $Sol_{CR}(\mathcal{R})$ we denote the set of all solutions of $CR(\mathcal{R})$.

Proposition 4 *For $\mathcal{R} = \{(B_1|A_1), \ldots, (B_n|A_n)\}$ let $(\kappa_1^-, \ldots, \kappa_n^-) \in Sol_{CR}(\mathcal{R})$. Then the function κ defined by (11) accepts $\mathcal{R}$.*

All c-representations built from (10) and (11) provide an excellent basis for model-based inference [KI02, KI01]. However, from the point of view of minimal specificity (see e.g. [BDP92]), those c-representations with minimal κ_i^- yielding minimal degrees of implausibility are most interesting.

We can define an obvious partial order on $Sol_{CR}(\mathcal{R})$ by defining

$$({\kappa'}_1^-, \ldots, {\kappa'}_n^-) \preceq (\kappa_1^-, \ldots, \kappa_n^-)$$

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iff $\kappa'^-_i \leqslant \kappa^-_i$ for $i \in \{1, \dots, n\}$. With $\prec$ we denote the irreflexive component of $\preceq$. Obviously, for any $(\kappa^-_1, \dots, \kappa^-_n) \in Sol_{CR}(\mathcal{R})$ there are infinitely many $(\kappa'^-_1, \dots, \kappa'^-_n) \in Sol_{CR}(\mathcal{R})$ with $(\kappa^-_1, \dots, \kappa^-_n) \preceq (\kappa'^-_1, \dots, \kappa'^-_n)$, e.g. by increasing the ranks of all already most implausible worlds by some fixed amount. As we are interested in minimal κ^-_i-vectors, an important question is whether there is always a unique minimal solution. This is not the case; the following example that is also discussed in [Mül04] illustrates that $Sol_{CR}(\mathcal{R})$ may have more than one minimal element.

Example 5 Let $\mathcal{R}_{birds} = \{R_1,\ R_2,\ R_3\}$ be the following set of conditionals:

$R_1:$	$(f\|b)$	*birds fly*
$R_2:$	$(a\|b)$	*birds are animals*
$R_3:$	$(a\|fb)$	*flying birds are animals*

From (10) we get

$$\begin{aligned} &\kappa^-_1 > 0 \\ &\kappa^-_2 > 0 - min\{\kappa^-_1,\ \kappa^-_3\} \\ &\kappa^-_3 > 0 - \kappa^-_2 \end{aligned}$$

and since $\kappa^-_i \geqslant 0$ according to (6), the two vectors

$$\begin{aligned} sol_1 &= (\kappa^-_1, \kappa^-_2, \kappa^-_3) = (1, 1, 0) \\ sol_2 &= (\kappa^-_1, \kappa^-_2, \kappa^-_3) = (1, 0, 1) \end{aligned}$$

are two different solutions of $CR(\mathcal{R}_{birds})$ that are minimal in $Sol_{CR}(\mathcal{R}_{birds})$ with respect to $\preceq$. ■

6 Conclusions and Further Work

Whereas in a probabilistic framework, the maximum entropy principle determines a unique model, there may be different minimal or "best" ranking functions for a set of qualitative rules $\mathcal{R}$. In general, finding a minimal ranking function involves an exponential search space since already the set Ω of all worlds is exponential in the number of propositional variables.

In [Bou99] an algorithm is given that computes a minimal OCF if such an OCF exists, but it is not able to find all minimal solutions if there are multiple minimal solutions. In [Mül04], an algorithm is developed returning all minimal c-representations; this algorithm is implemented in the object-oriented programming language C#.

As demonstrated above, the set of all c-representations for $\mathcal{R}$ correspond to the solutions of the constraint satisfaction problem $CR(\mathcal{R})$, where $CR(\mathcal{R})$ is a direct and declarative specification of the solution space. Thus, instead of using an imperative programming language approach as in [Mül04], it seems an obvious and natural choice to use constraint programming techniques for solving $CR(\mathcal{R})$, taking into account the additional constraint

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that the solutions of interest are the minimal solutions in $Sol_{CR}(\mathcal{R})$. Due to the problem's complexity, the efficient solving of $CR(\mathcal{R})$ poses a challenge for constraint programming.

By using declarative constraint programming techniques, we additionally expect advantages with respect to modifications of the set of constraints. For instance, one might want to use an ordering of the solution space that is different from the component-wise ordering $\preceq$ employed above in Section 4. Furthermore, it is an open problem how to strengthen the constraints defining a c-representation so that a unique solution is guaranteed to exist. In both cases, the declarative nature of constraint programming might allow for an easy modification of a solution finding method taking into account a modified constraint system.

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