

Polynomial expansion and sublinear separators

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Thursday 4th May, 2017

Join work with Louis Esperet (G-SCOP, Grenoble).

Theorem (Plotkin, Rao, and Smith, SODA 1994)

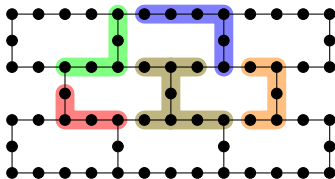
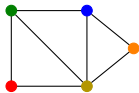
polynomial expansion \Rightarrow *sublinear separators*

Theorem (Dvořák and Norin, SIDMA 2016)

sublinear separators \Rightarrow *polynomial expansion*.

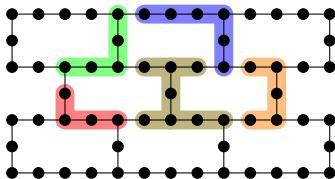
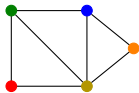
(Bounded-depth) minors

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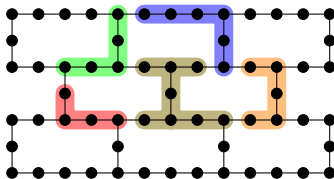
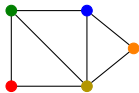
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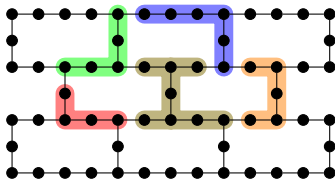
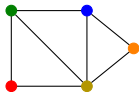
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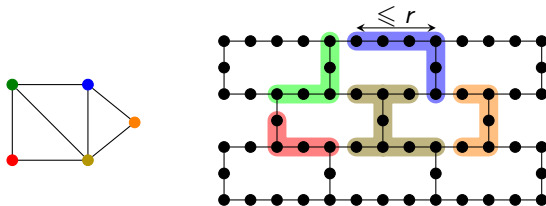
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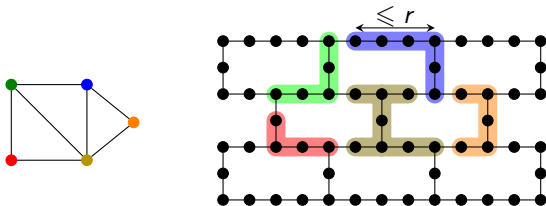


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Ex: a 0-minor is a subgraph.

- r -minor of G : obtained by contracting disjoint subgraphs of radius $\leq r$ in a subgraph of G ;

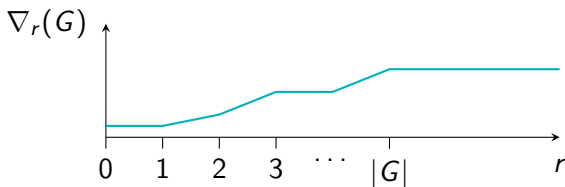
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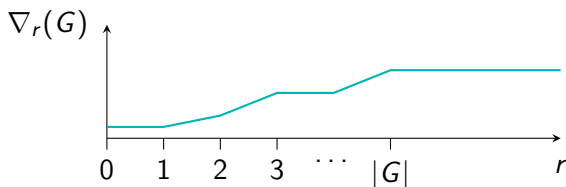
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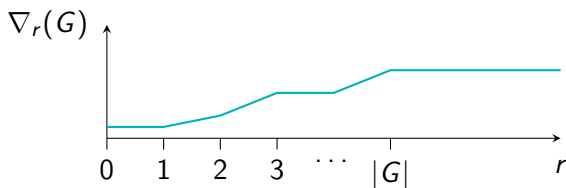
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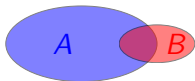


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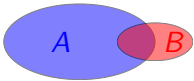
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- \mathcal{C} has **polynomial expansion** if f is polynomial.

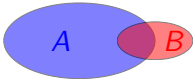
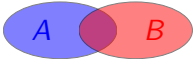
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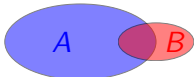
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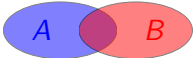
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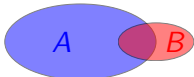
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- **sublinear separator**: separator of size $O(n^\beta)$ with $0 \leq \beta < 1$.

Ex: planar graphs have balanced separators of size $O(n^{1/2})$.

Why do we care about (small) separators?

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Theorem (Dvořák and Norin, 2014)

Let $\beta \in [0, 1)$.

$\forall H \subseteq G$, H has a balanced separator of order $\leq c|H|^\beta$

\Downarrow
 $\forall H \subseteq G$, $\mathbf{tw}(H) \leq 105c|H|^\beta$.

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Separators & expansion

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\mathcal{C} has *expansion* $O(r^{1/\delta})$



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Theorem (Esperet and R., 2017)

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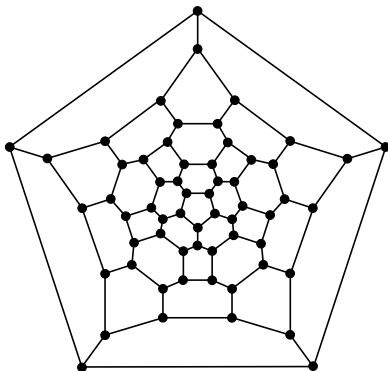


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A few words about expanders

Expander: **small** subsets of V have **linearly many** neighbors at least.

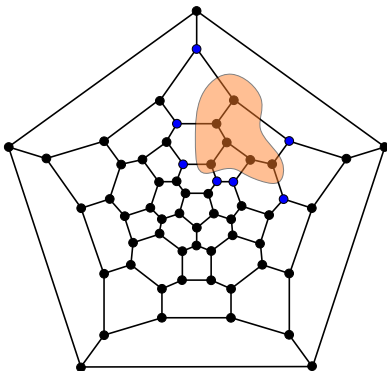
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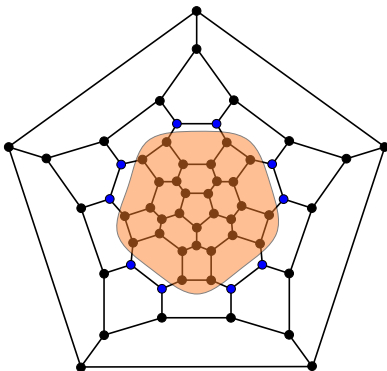
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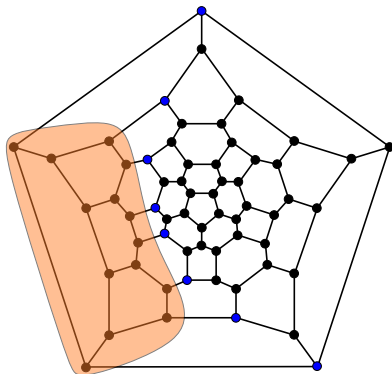
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Expanders have linear treewidth

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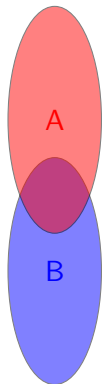
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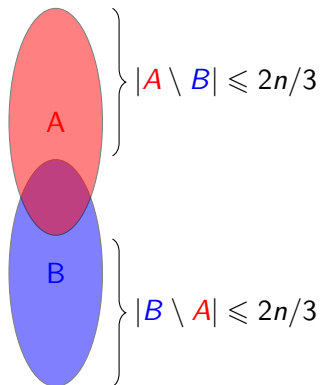
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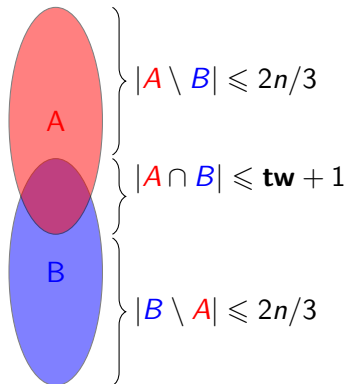
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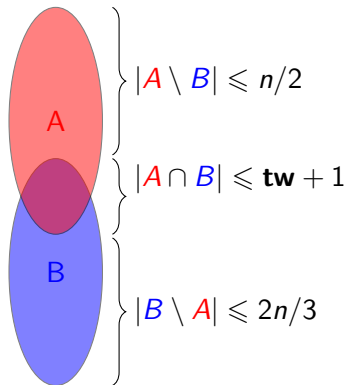
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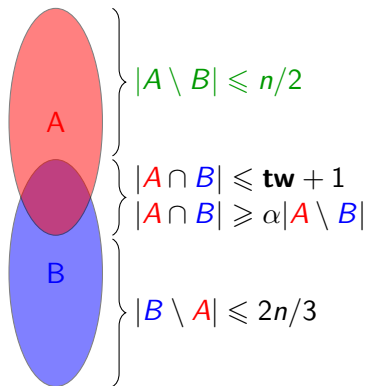
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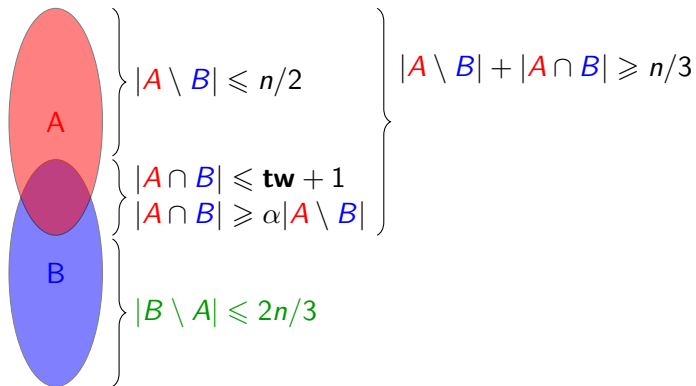
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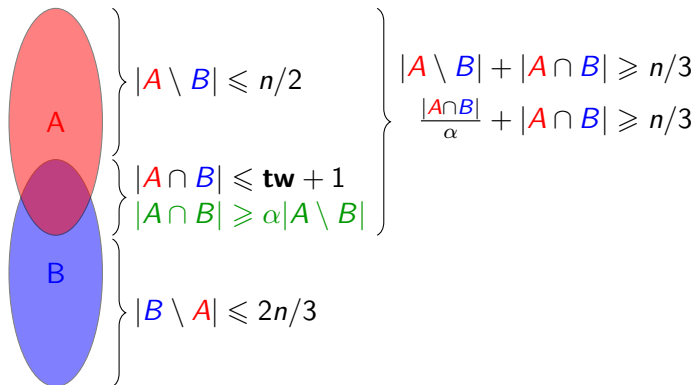
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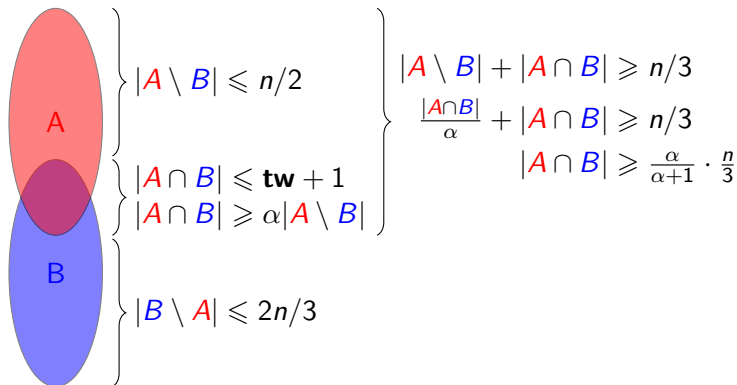
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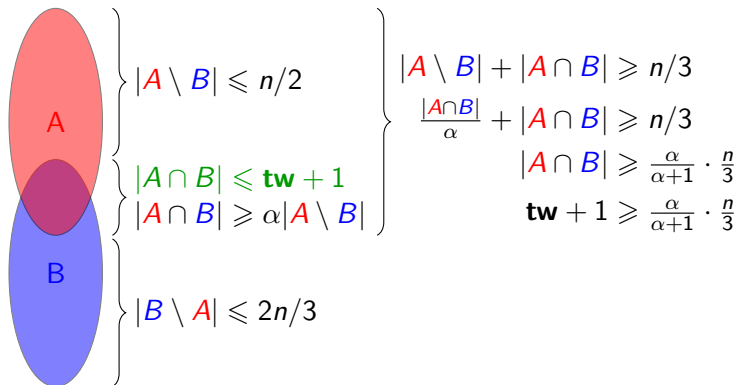
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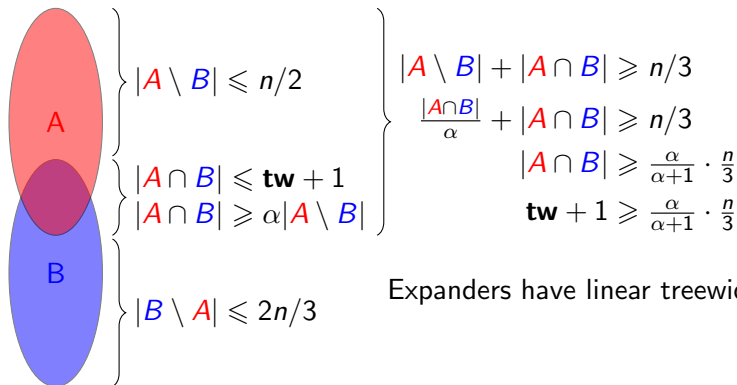
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Expanders have linear treewidth!

Theorem (Shapira and Sudakov, Combinatorica 2015)

Any graph G contains a subgraph H s.t.:

- 1 $d(H) \geq 0.9 \cdot d(G)$;
- 2 H is an $\frac{1}{\text{polylog } |H|}$ -expander.

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Corollary

Any graph G contains a subgraph H with

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Sketch of the proof

Current goal

\mathcal{C} has **balanced sep. of order** $O(n^{1-\delta}) \Rightarrow \mathcal{C}$ has **expansion** $O(r^{c/\delta})$

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Lemma (expander lemma)

Every F has a sgr. H with $d \geq 0.9d(F)$ and $\mathbf{tw} = \Omega\left(\frac{|H|}{\text{polylog}|H|}\right)$.

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Theorem (Chekuri and Chuzhoy, SODA 2015)

Every G with $\mathbf{tw} = k$ has a sgr. with $\Delta \leq 3$ and $\mathbf{tw} \geq \frac{k}{\text{polylog } k}$.

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G' sgr. of G corresponding to H' : $\mathbf{tw}(G') = \Omega\left(\frac{|H|}{\text{polylog}|H|}\right)$

Current goal

$$d(H) = O(r^{c/\delta})$$

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Theorem (Dvořák and Norin, 2014)

Balanced separators of order $O(n^{1-\delta}) \Rightarrow \mathbf{tw} = O(n^{1-\delta})$

Sketch of the proof

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Thank you!